Applications of Statistical Catch-at-Age Assessment Methodology to Gulf of Maine cod

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Summary

The Gulf of Maine cod SCAA assessment presented to the 2008 GARM III meetings is updated to take account of new data. Two Base Cases are put forward: one involving a Ricker form for the stock-recruitment relationship, and the other treating expected recruitment as effectively independent of spawning biomass. The former provides a better fit to the spawning biomass and recruitment estimates, but is less robust by way of lesser precision and a strong retrospective pattern. Of a variety of sensitivity tests, increasing *M* from 0.2 to 0.3 to lessen the dome in the estimated selectivity-at-age vector leads to an improved likelihood and a resource estimated to be closer to its MSY biomass.

Introduction

This paper updates the assessment of the Gulf of Maine cod stock using essentially the same methodology as in previous SCAA assessments of this resource presented to the 2008 GARM III meetings (e.g. Butterworth and Rademeyer, 2008). A number of sensitivities to various choices for inputs are also reported.

Data and Methodology

The catch and survey based data (including catch-at-length information) and some biological data are listed in Tables in Appendix A.

The details of the SCAA assessment methodology are provided in Appendix B. Table 1 gives a full list of symbols used in the Tables following, together with their definitions.

Results and Discussion

Base Cases

Results are reported for two Base Cases and a number of sensitivities. Base Case A makes use of a Ricker form for the spawning stock recruitment relationship, while a Beverton-Holt form with fixed steepness (*h*=0.98, basically reflecting a constant expected recruitment scenario) is used for Base Case B. Tables 2 and 3 give results of management quantities for these two Base Cases and their sensitivities. Spawning biomass trajectories are plotted for the two Base Cases in Fig. 1, while the fits to the surveys, CPUE and commercial and survey catch-at-age information are shown in Fig. 2. Fig. 3 plots the spawning stock recruitment relationships as well as the estimated recruitment residuals for the two Base Cases. The selectivities-at-age estimated are shown in Fig. 4 and compared to the GARM-III VPA selectivities.

Five year retrospective analyses were conducted for the Base Cases and the results are given in Table 4 and Fig. 5.

Selectivity at older ages

Sensitivity of the results to the assumption about the selectivity above the data plus-group (9+) have been investigated (Cases A/B1a, b). For the Base Cases, the slope estimated between ages 8 and 9 is assumed to continue exponentially to the model plus group (11+). For Cases A/B1a, the selectivity is assumed to be flat from age 9, while for cases A/B1b, the estimated slope between ages 8 and 9 is doubled. The resulting selectivities are shown in Fig. 6 for Base Case A.

Natural mortality

Natural mortality in the Base Cases is fixed at 0.2 for all ages. In cases A/B2a, b the natural mortality is increased to 0.3 and to 0.4 respectively, while in cases A/B2c, d, it is assumed to increase linearly from 0.2 (0.3 for A/B2d) for ages 4 and below to 0.4 for age 11+. The level of natural mortality affects the estimated fishing selectivities, so that these are compared for the NEFSC surveys and commercial fleet for Base Case A in Fig. 7.

Starting year

The starting year for the Base Cases is 1964. A sensitivity to this has been run by starting the model in 1982. This is Case A/B3.

Excluding LPUE

The effect of excluding the LPUE series in the model fit is investigated in cases A/B4.

Initial conditions

Sensitivities to different initial (1964) conditions (θ and ϕ) for each Base Case have been conducted and results are given in Table 4.

Overall

Spawning biomass trajectories for both Base Cases and sensitivities (to Base Case A) are compared in Fig. 8.

Discussion

The selection of a Base Case is not straightforward. Certainly the stock-recruitment data prefer a Ricker-like relationship as recruitment estimates tend to be lower at higher spawning biomasses (Fig. 3). However the associated assessment shows a strong retrospective pattern (Table 4a and Fig. 5a). This pattern is greatly ameliorated by use instead of a Beverton-Holt form with fixed steepness (at h = 0.98 – see Table 4b and Fig. 5b) – in practical terms no dependence of recruitment on spawning biomass. Both forms show similarly good fits to the data (Fig. 2) – estimates of autocorrelation in residual series are small, so that the likelihood has not taken this possibility into account. The only reason that the overall penalised negative log likelihood favours the Ricker of these two is its better fit compared to the constant expected recruitment (see Tables 2 and 3).

The primary feature in the data that is leading to these results is the strong decline in survey estimates for the NEFSC spring survey over the last five years (see Fig. 2, Table A.5), which also results in diminished recruitment estimates for the most recent years.

In essence there is a trade-off choice to be made of the better fit of the Ricker model against the greater robustness (better precision, less retrospective pattern) of what amounts to a constant expected recruitment model. For completeness we retain both of these assessments as Base Cases, and have reported sensitivity results for both. However for Beverton-Holt with h = 0.98, the estimates of B_{MSY} become inappropriately low, so that it is likely better to use a proxy for B_{MSY} in this case.

These Base Case assessments show markedly domed selectivity (both NEFSC survey and commercial) as a result of the low proportions of higher ages caught in both operations (Fig. 4). The extent of this dome can be ameliorated by either or both of increasing the age-independent value for M or allowing M to increase with age. Results for such sensitivities are shown in Tables 2 and 3, while Figs 4 and 8 show that these changes do impact both selectivity patterns and historic trends in spawning biomass. Such higher values of M indicate a resource currently closer to B_{MSY} , and for M= 0.3 lead to fits to the data that are to be preferred to those for M = 0.2 in terms of AIC.

Other results of note are first that the data certainly prefer estimation θ and ϕ (Table 5). Commencing the assessment later (in 1982 rather than 1964) certainly impacts precision of estimates, which reduce substantially for Case A (the Ricker relationship), though less so for Base Case B.

Acknowledgements

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References

Butterworth DS and Rademeyer RA. 2008. Statistical catch-at-age analysis vs ADAPT-VPA: the case of Gulf of Maine cod. GARM-III Working paper 2.2a.

Table 1: Definitions of symbols used in presenting results (the order follows that used for Tables 2 and 3). Unless otherwise indicated biomasses are "deterministic", i.e. as estimated in the model fit, prior to any bias adjustment for the lognormal recruitment variability assumed.

-lnL: overall	Total negative log-likelihood
-lnL: survey/comCAA /survCAA/RecRes	Contribution to -lnL from survey indices/commercial catch-at-age proportions/survey catch-at-age proportions/recruitment residuals
h	Stock recruitment curve steepness
γ	Parameter of generalised Ricker S/R function (γ=1 for Ricker)
θ	B^{sp}/K^{sp} for starting year
ϕ	$Z_a \sim M_a + \phi$ for starting year
K^{sp}	Pristine spawning biomass
B^{sp}_{2010}	Spawning biomass in 2010
$MSYL^{sp}$	B^{sp}_{MSY}/K_{sp}
B^{sp}_{MSY}	Spawning biomass at MSY
MSY	Maximum sustainable yield
F_{MSY}	Fishing mortality rate (F) at MSY (corresponds to F at the age at which commercial selectivity = 1)
F_{2010}	F for year 2010
F_{2010} (av ages 4-5)	Average of F on ages 4 and 5 in 2010
$F_{40\%}$	F at which B^{sp}/R (R=recruitment) equals 40% of its value when F=0
$B^{sp}_{MSY\_40\%}$	Spawning biomass corresponding to F40%; evaluated as (B^{sp}/R for $F_{40\%}$)R_{av} where R_{av} is average of recruitment estimates over the whole period (1964-2010)
MSY_40%	MSY corresponding to F40%; evaluated as (Y/R for $F_{40\%}$)R_{av}
C_{2011}	Catch in 2011, assumed equal to 2010 catch
$C_{2012}(F_{MSY})$	Projected 2012 catch under F_{MSY}
$C_{2012}(F_{status\ quo})$	Projected 2012 catch under F_{2012}=F_{2008}
$C_{2012}(F_{rebuild})$	Projected 2012 catch under $F_{rebuild}$
q spring/autumn	Multiplicative bias for spring/autumn NEFSC survey swept-area-based biomass estimate relative to actual survey selectivity-at-age weighted biomass
Slope_com/surv 7/8	Selectivity slope given by $S_8 = e^{-Slope} S_7$
σ_R_out	Standard deviation of dsitribution of logs of multiplicative recruitment residuals about estiamted S/R relationship
M0/M11+	Natural mortality rate for age 0/11+
q	Multiplicative bias for survey swept-area-based biomass estimate relative to actual survey selectivity-at-age weighted biomass
σ_{Add}	Additional variance for survey series

Table 2: Estimates of management quantities for the Gulf of Maine cod with a Ricker stock-recruitment curve. Values in bold are inputs, and those in parentheses are Hessian based CV's. Mass units are '000 tons.

	A0		A1a		A1b		A2a		A2b		A2c		A2d		A3		A4		A6	
	Base Case A Ricker		NEFSC and comm sel flat from age 9		NEFSC and comm sel slope x 2 from age 9		*M* =0.3		*M* =0.4		*M* increasing from 0.2 at age 4		*M* changing from 0.3 at age 4		Start in 1982		Exclude LPUE		σ_R =0.6	
-lnL: overall	113.3		114.2		113.3		111.3		114.6		113.0		112.0		188.2*		121.7*		85.8*	
-lnL: survey	-11.3		-10.8		-11.5		-13.4		-14.3		-13.7		-14.2		10.3		-2.1		-11.1	
-lnL: comCAA	-87.4		-86.4		-87.6		-88.1		-88.7		-87.8		-88.4		-58.5		-89.3		-87.4	
-lnL: survCAA	167.1		166.5		167.5		168.9		173.4		169.5		170.5		214.3		168.3		160.7	
-lnL: RecRes	44.9		45.0		44.9		43.9		44.1		45.1		44.2		22.0		44.9		23.5	
h	1.28	(0.16)	1.32	(0.16)	1.26	(0.16)	0.99	(0.13)	0.74	(0.13)	1.12	(0.14)	0.91	(0.12)	0.62	(0.20)	1.25	(0.16)	1.38	(0.19)
γ	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-
θ	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.47	(0.54)	0.95	(0.00)	0.95	(0.00)
ϕ	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-
K^{sp}	182.8	(0.15)	176.5	(0.15)	186.6	(0.15)	125.1	(0.12)	106.6	(0.12)	116.2	(0.14)	110.0	(0.11)	495.6	(0.54)	187.1	(0.15)	185.7	(0.17)
B^{sp}_{2010}	41.5	(0.17)	41.2	(0.17)	42.0	(0.17)	37.3	(0.14)	37.0	(0.14)	34.0	(0.16)	35.6	(0.14)	96.2	(0.16)	42.5	(0.17)	40.0	(0.18)
B^{sp}_{2010}/K	0.23	(0.12)	0.23	(0.12)	0.23	(0.12)	0.30	(0.13)	0.35	(0.13)	0.29	(0.13)	0.32	(0.14)	0.19	(0.57)	0.23	(0.12)	0.22	(0.12)
$MSYL^{sp}$	0.34	(0.14)	0.34	(0.14)	0.34	(0.14)	0.35	(0.13)	0.36	(0.13)	0.35	(0.13)	0.36	(0.14)	0.35	(0.34)	0.34	(0.14)	0.34	(0.17)
B^{sp}_{MSY}	62.5	(0.10)	60.3	(0.10)	63.9	(0.10)	43.8	(0.10)	38.3	(0.10)	40.9	(0.10)	39.2	(0.10)	171.0	(0.28)	64.2	(0.10)	62.7	(0.13)
$B^{sp}_{2010}/B^{sp}_{MSY}$	0.66	(0.16)	0.68	(0.15)	0.66	(0.16)	0.85	(0.14)	0.97	(0.14)	0.83	(0.15)	0.91	(0.15)	0.56	(0.33)	0.66	(0.16)	0.64	(0.17)
MSY	10.3	(0.08)	10.3	(0.08)	10.3	(0.09)	9.7	(0.10)	9.1	(0.10)	10.1	(0.09)	9.6	(0.10)	9.2	(0.29)	10.3	(0.09)	11.3	(0.12)
F_{MSY}	0.29	(0.00)	0.30	(0.00)	0.29	(0.00)	0.27	(0.00)	0.25	(0.00)	0.29	(0.00)	0.27	(0.00)	0.15	(0.00)	0.29	(0.00)	0.31	(0.00)
F_{2010}	0.57	(0.17)	0.57	(0.17)	0.57	(0.17)	0.50	(0.17)	0.46	(0.17)	0.53	(0.17)	0.49	(0.16)	0.36	(0.19)	0.56	(0.17)	0.60	(0.18)
F_{2010} (av ages 4-5)	0.57	(0.17)	0.57	(0.17)	0.57	(0.17)	0.50	(0.17)	0.45	(0.17)	0.53	(0.17)	0.49	(0.16)	0.36	(0.19)	0.56	(0.17)	0.60	(0.18)
$F_{40\%}$	0.20	(0.00)	0.20	(0.00)	0.20	(0.00)	0.23	(0.00)	0.28	(0.00)	0.23	(0.00)	0.25	(0.00)	0.22	(0.00)	0.20	(0.00)	0.20	(0.00)
$B^{sp}_{MSY\_40\%}$	81.7	(0.06)	80.8	(0.06)	82.2	(0.06)	47.1	(0.05)	33.6	(0.05)	47.3	(0.05)	39.1	(0.05)	98.6	(0.06)	82.0	(0.06)	81.0	(0.06)
MSY_40%	7.9	(0.02)	7.8	(0.02)	7.9	(0.02)	8.3	(0.02)	9.3	(0.02)	8.4	(0.02)	8.7	(0.02)	8.8	(0.05)	7.9	(0.02)	7.8	(0.02)
C_{2011}	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-
$C_{2012}(F_{MSY})$	5.5	(0.25)	5.5	(0.24)	5.4	(0.25)	5.8	(0.23)	5.8	(0.23)	5.9	(0.24)	5.9	(0.22)	4.7	(0.24)	5.5	(0.25)	5.2	(0.27)
$C_{2012}(F_{status\ quo})$	8.6	(0.08)	8.6	(0.08)	8.7	(0.08)	8.8	(0.07)	8.8	(0.07)	8.8	(0.08)	8.8	(0.07)	9.1	(0.06)	8.7	(0.08)	8.3	(0.10)
Slope_com 7/8	0.49	(0.11)	0.50	(0.11)	0.50	(0.11)	0.36	(0.11)	0.22	(0.11)	0.38	(0.11)	0.31	(0.10)	0.65	(0.13)	0.49	(0.11)	0.48	(0.12)
Slope_surv 7/8	0.65	(0.11)	0.63	(0.11)	0.66	(0.11)	0.51	(0.10)	0.28	(0.10)	0.55	(0.11)	0.44	(0.10)	0.78	(0.11)	0.65	(0.11)	0.65	(0.12)
σ_{R}_out	0.55	(0.04)	0.55	(0.04)	0.55	(0.04)	0.55	(0.04)	0.55	(0.04)	0.55	(0.04)	0.55	(0.04)	0.49	(0.05)	0.55	(0.04)	0.60	(0.05)
M0-M4	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.20**		**0.30**		**0.20**		**0.20**		**0.20**	
M5	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.23**		**0.31**		**0.20**		**0.20**		**0.20**	
M6	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.26**		**0.33**		**0.20**		**0.20**		**0.20**	
M7	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.29**		**0.34**		**0.20**		**0.20**		**0.20**	
M8	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.31**		**0.36**		**0.20**		**0.20**		**0.20**	
M9	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.34**		**0.37**		**0.20**		**0.20**		**0.20**	
M10	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.37**		**0.39**		**0.20**		**0.20**		**0.20**	
M11+	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.40**		**0.40**		**0.20**		**0.20**		**0.20**	
	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}
NEFSC spring	0.222	0.17	0.224	0.17	0.220	0.17	0.202	0.16	0.187	0.16	0.207	0.16	0.195	0.16	0.137	0.20	0.218	0.17	0.227	0.16
NEFSC fall	0.187	0.11	0.189	0.11	0.185	0.11	0.170	0.10	0.157	0.09	0.174	0.10	0.163	0.10	0.105	0.10	0.183	0.11	0.190	0.11
MADMF spring	0.631	0.10	0.633	0.10	0.630	0.10	0.408	0.10	0.261	0.10	0.620	0.10	0.405	0.10	0.502	0.12	0.629	0.10	0.648	0.10
MADMF fall	0.127	0.50	0.128	0.50	0.127	0.50	0.082	0.50	0.052	0.50	0.122	0.50	0.080	0.50	0.100	0.50	0.126	0.50	0.136	0.50
LPUE	0.008	0.05	0.008	0.05	0.008	0.05	0.007	0.05	0.006	0.06	0.008	0.05	0.007	0.06	0.005	0.05	0.008	0.05	0.008	0.05

Table 3: Estimates of management quantities for the Gulf of Maine cod with a Beverton-Holt stock-recruitment curve with steepness *h* fixed at 0.98. Values in bold are inputs, and those in parentheses are Hessian based CV's. Mass units are '000 tons.

	B0		B1a		B1b		B2a		B2b		B2c		B2d		B3		B4		B6	
	Base Case B Beverton-Holt		NEFSC and comm sel flat from age 9		NEFSC and comm sel slope x 2 from		M=0.3		M=0.4		M increasing from 0.2 at age 4		M changing from 0.3 at age 4		Start in 1982		Exclude LPUE		σ_R=0.6	
-lnL: overall	121.8		123.1		121.7		116.0		116.9		120.2		115.8		190.6*		130.0*		98.1*	
-lnL: survey	-11.8		-11.4		-12.0		-13.2		-13.6		-13.2		-13.9		10.6		-2.8		-10.2	
-lnL: comCAA	-87.0		-86.0		-87.2		-88.7		-88.7		-87.4		-88.8		-58.5		-88.8		-87.6	
-lnL: survCAA	169.7		169.1		170.0		169.5		172.8		170.0		170.4		214.5		170.8		168.3	
-lnL: RecRes	51.0		51.4		50.9		48.4		46.4		50.7		48.1		24.0		50.8		27.5	
h	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-	**0.98**	-
γ	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-	**1.00**	-
θ	0.94	(0.17)	0.90	(0.16)	0.94	(0.16)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.95	(0.00)	0.72	(0.09)	0.95	(0.16)	0.15	(0.19)
ϕ	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.10**	-	**0.50**	-
K^{sp}	250.2	(0.08)	246.4	(0.07)	251.2	(0.08)	145.2	(0.08)	105.4	(0.07)	181.1	(0.07)	120.9	(0.07)	334.6	(0.10)	251.1	(0.08)	258.4	(0.10)
B^{sp}_{2010}	55.6	(0.17)	54.2	(0.17)	55.9	(0.17)	42.6	(0.14)	39.1	(0.14)	45.9	(0.14)	38.9	(0.14)	108.7	(0.16)	56.4	(0.17)	45.9	(0.19)
B^{sp}_{2010}/K	0.22	(0.15)	0.22	(0.15)	0.22	(0.15)	0.29	(0.11)	0.37	(0.11)	0.25	(0.11)	0.32	(0.11)	0.32	(0.13)	0.22	(0.15)	0.18	(0.17)
$MSYL^{sp}$	0.10	(0.04)	0.10	(0.04)	0.10	(0.03)	0.10	(0.02)	0.12	(0.04)	0.11	(0.02)	0.12	(0.02)	0.09	(0.03)	0.10	(0.03)	0.10	(0.04)
B^{sp}_{MSY}	25.5	(0.10)	25.5	(0.09)	25.5	(0.09)	15.1	(0.09)	12.9	(0.08)	20.1	(0.09)	14.0	(0.08)	30.6	(0.11)	25.6	(0.10)	26.9	(0.12)
$B^{sp}_{2010}/B^{sp}_{MSY}$	2.18	(0.13)	2.13	(0.13)	2.19	(0.13)	2.82	(0.11)	3.03	(0.12)	2.29	(0.11)	2.79	(0.11)	3.55	(0.13)	2.20	(0.13)	1.71	(0.15)
MSY	9.2	(0.07)	9.1	(0.07)	9.3	(0.07)	9.9	(0.08)	11.4	(0.07)	9.3	(0.07)	10.2	(0.07)	11.0	(0.10)	9.3	(0.07)	9.6	(0.10)
F_{MSY}	0.51	(0.00)	0.50	(0.00)	0.51	(0.00)	0.64	(0.00)	0.92	(0.00)	0.53	(0.00)	0.68	(0.00)	0.54	(0.00)	0.51	(0.00)	0.49	(0.00)
F_{2010}	0.47	(0.17)	0.48	(0.17)	0.47	(0.17)	0.45	(0.16)	0.43	(0.17)	0.47	(0.16)	0.45	(0.16)	0.30	(0.17)	0.46	(0.17)	0.55	(0.18)
F_{2010} (av ages 4-5)	0.47	(0.17)	0.48	(0.17)	0.47	(0.17)	0.45	(0.16)	0.42	(0.16)	0.47	(0.16)	0.45	(0.16)	0.30	(0.17)	0.46	(0.17)	0.55	(0.18)
$F_{40\%}$	0.21	(0.00)	0.21	(0.00)	0.21	(0.00)	0.23	(0.00)	0.28	(0.00)	0.22	(0.00)	0.25	(0.00)	0.22	(0.00)	0.21	(0.00)	0.20	(0.00)
$B^{sp}_{MSY\_40\%}$	87.1	(0.06)	85.7	(0.06)	87.5	(0.06)	48.6	(0.05)	33.7	(0.05)	62.0	(0.04)	39.9	(0.04)	102.6	(0.06)	87.3	(0.06)	83.7	(0.07)
MSY_40%	8.1	(0.02)	8.1	(0.02)	8.2	(0.02)	8.5	(0.02)	9.3	(0.03)	8.4	(0.02)	8.8	(0.02)	9.1	(0.05)	8.2	(0.02)	8.0	(0.02)
C_{2011}	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-	11.4	-
$C_{2012}(F_{MSY})$	10.4	(0.22)	10.1	(0.22)	10.6	(0.22)	12.9	(0.21)	17.2	(0.20)	10.7	(0.22)	13.5	(0.21)	16.7	(0.20)	10.6	(0.22)	8.2	(0.26)
$C_{2012}(F_{status\ quo})$	8.9	(0.06)	8.8	(0.06)	9.0	(0.06)	8.9	(0.06)	9.0	(0.05)	8.9	(0.06)	8.9	(0.06)	9.4	(0.05)	9.0	(0.06)	8.4	(0.09)
Slope_com 7/8	0.54	(0.11)	0.56	(0.11)	0.55	(0.11)	0.39	(0.09)	0.22	(0.09)	0.49	(0.09)	0.33	(0.09)	0.64	(0.13)	0.54	(0.11)	0.53	(0.12)
Slope_surv 7/8	0.68	(0.10)	0.67	(0.10)	0.69	(0.10)	0.56	(0.09)	0.27	(0.09)	0.63	(0.07)	0.48	(0.09)	0.78	(0.11)	0.68	(0.10)	0.61	(0.11)
σ_R_out	0.59	(0.04)	0.59	(0.04)	0.59	(0.04)	0.57	(0.04)	0.56	(0.04)	0.59	(0.04)	0.57	(0.04)	0.52	(0.05)	0.59	(0.04)	0.65	(0.04)
M0-M4	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.20**		**0.30**		**0.20**		**0.20**		**0.20**	
M5	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.21**		**0.31**		**0.20**		**0.20**		**0.20**	
M6	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.23**		**0.33**		**0.20**		**0.20**		**0.20**	
M7	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.24**		**0.34**		**0.20**		**0.20**		**0.20**	
M8	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.26**		**0.36**		**0.20**		**0.20**		**0.20**	
M9	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.27**		**0.37**		**0.20**		**0.20**		**0.20**	
M10	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.29**		**0.39**		**0.20**		**0.20**		**0.20**	
M11+	**0.20**		**0.20**		**0.20**		**0.30**		**0.40**		**0.30**		**0.40**		**0.20**		**0.20**		**0.20**	
	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}	$q \times 10^3$	σ_{Add}
NEFSC spring	0.185	0.17	0.188	0.17	0.184	0.17	0.183	0.17	0.187	0.16	0.183	0.17	0.182	0.17	0.129	0.21	0.183	0.17	0.206	0.17
NEFSC fall	0.157	0.11	0.160	0.11	0.156	0.10	0.153	0.10	0.157	0.10	0.155	0.10	0.153	0.10	0.099	0.11	0.155	0.11	0.181	0.11
MADMF spring	0.591	0.10	0.596	0.10	0.589	0.10	0.395	0.10	0.258	0.10	0.594	0.10	0.397	0.10	0.478	0.12	0.588	0.10	0.631	0.10
MADMF fall	0.119	0.50	0.121	0.50	0.119	0.50	0.079	0.50	0.051	0.50	0.118	0.50	0.078	0.50	0.096	0.50	0.118	0.50	0.132	0.50
LPUE	0.007	0.05	0.008	0.05	0.007	0.05	0.007	0.05	0.007	0.06	0.007	0.05	0.007	0.06	0.005	0.05	0.007	0.05	0.008	0.05

Table 4a: Estimates of management quantities for the retrospective analysis for Base Case A (Ricker).

Retrospective for Base Case A (Ricker)					
	2009	2008	2007	2006	2005
K^{sp}	174.3	164.3	151.3	142.6	133.6
$MSYL^{sp}$	0.34	0.33	0.33	0.32	0.32
B^{sp}_{MSY}	58.7	54.5	49.6	46.1	42.5
MSY	10.9	11.4	11.4	11.7	12.0
B^{sp}_{MSY}_40%	84.3	86.3	86.1	87.1	87.7
MSY_40%	8.1	8.4	8.4	8.6	8.7

Table 4b: Estimates of management quantities for the retrospective analysis for Base Case B (Beverton-Holt, *h*=0.98).

Retrospective for Base Case B (Beverton-Holt, *h* =0.98)					
	2009	2008	2007	2006	2005
K^{sp}	260.3	262.9	258.6	261.5	262.8
$MSYL^{sp}$	0.10	0.10	0.10	0.10	0.10
B^{sp}_{MSY}	26.6	27.0	26.8	27.2	27.4
MSY	9.6	9.7	9.6	9.7	9.7
B^{sp}_{MSY}_40%	89.8	90.8	90.2	91.2	91.0
MSY_40%	8.4	8.5	8.5	8.6	8.6

Table 5a: Estimates of management quantities for Base Case A (Ricker stock-recruit relationship) with different initial conditions (θ and ϕ).

		θ estimated	θ=0.75	θ=0.5
ϕ=0.1	-lnL: overall	121.7	128.8	143.0
	θ	0.95	0.75	0.50
	h	1.25	1.19	0.92
	K^{sp}	187.1	195.5	277.0
	B^{sp}_{2010}/K	0.23	0.19	0.14
	$B*^{sp}_{MSY}$	64.2	67.6	99.0
	$B^{sp}_{2010}/B^{sp}_{MSY}$	0.66	0.56	0.39
ϕ=0.3	-lnL: overall	129.9	135.7	130.1
	θ	0.54	0.75	0.50
	h	1.06	0.94	1.05
	K^{sp}	202.7	227.0	203.2
	B^{sp}_{2010}/K	0.26	0.29	0.25
	$B*^{sp}_{MSY}$	71.0	80.5	71.3
	$B^{sp}_{2010}/B^{sp}_{MSY}$	0.74	0.83	0.70
ϕ=0.5	-lnL: overall	134.0	186.9	159.4
	θ	0.19	0.75	0.50
	h	0.91	0.49	0.77
	K^{sp}	232.7	318.7	251.7
	B^{sp}_{2010}/K	0.22	0.34	0.32
	$B*^{sp}_{MSY}$	83.0	114.8	90.8
	$B^{sp}_{2010}/B^{sp}_{MSY}$	0.62	0.93	0.89

Table 5b: Estimates of management quantities for Base Case B (Beverton-Holt, h=0.98 stock-recruit relationship) with different initial conditions (θ and ϕ).

		θ estimated	θ=0.75	θ=0.5
ϕ=0.1	-lnL: overall	130.0	131.0	137.1
	θ	0.95	0.75	0.50
	h	0.98	0.98	0.98
	K^{sp}	251.1	250.5	247.6
	B^{sp}_{2010}/K	0.22	0.19	0.15
	$B*^{sp}_{MSY}$	25.6	26.1	27.2
	$B^{sp}_{2010}/B^{sp}_{MSY}$	2.20	1.87	1.33
ϕ=0.3	-lnL: overall	132.4	137.9	132.7
	θ	0.44	0.75	0.50
	h	0.98	0.98	0.98
	K^{sp}	250.9	253.6	251.3
	B^{sp}_{2010}/K	0.22	0.30	0.24
	$B*^{sp}_{MSY}$	25.7	25.1	25.4
	$B^{sp}_{2010}/B^{sp}_{MSY}$	2.19	3.03	2.39
ϕ=0.5	-lnL: overall	137.4	171.7	155.0
	θ	0.19	0.75	0.50
	h	0.98	0.98	0.98
	K^{sp}	252.0	253.0	250.7
	B^{sp}_{2010}/K	0.23	0.40	0.35
	$B*^{sp}_{MSY}$	25.7	24.4	24.4
	$B^{sp}_{2010}/B^{sp}_{MSY}$	2.22	4.16	3.58

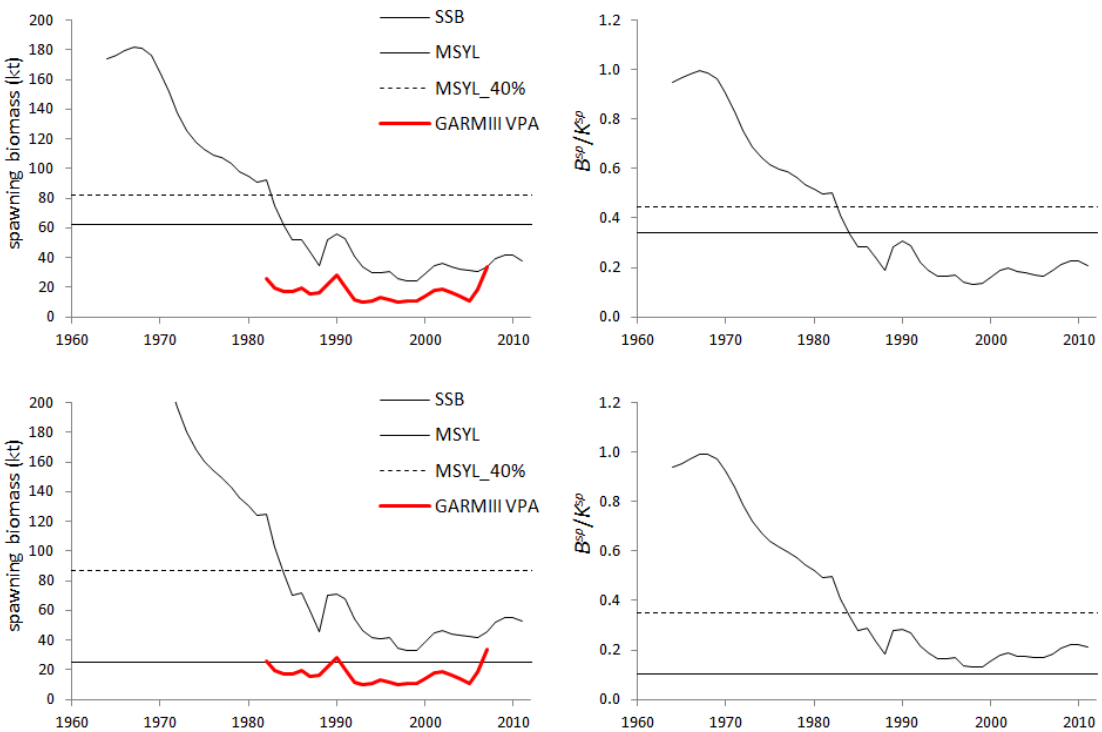

Fig. 1: Spawning biomass trajectories, both in absolute term and relative to pre-exploitation level, for the **Base Case A (Ricker)** (top row) and **Base Case B (Beverton-Holt, *h*=0.98)** (bottom row). The MSYL and 2008 VPA GARM III spawning biomass trajectories are also shown.

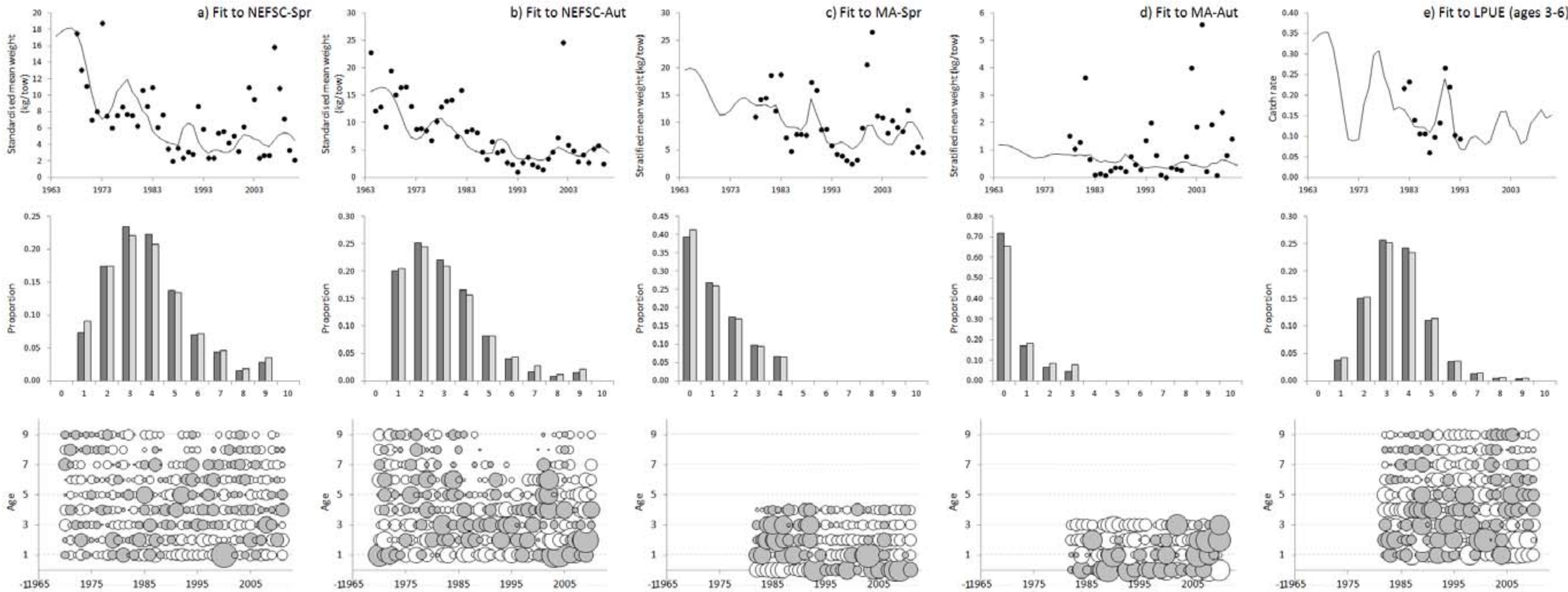

Fig. 2a: Fit to the abundance indices for **Base Case A (Ricker)** (top row) and to the survey and commercial catch-at-age data. The middle row plots compare the observed and predicted CAA as averaged over all years for which data are available, while the bottom row plots show the standardised residuals, with the size (area) of the bubbles being proportional to the magnitude of the corresponding standardised residuals. For positive residuals, the bubbles are grey, whereas for negative residuals, the bubbles are white.

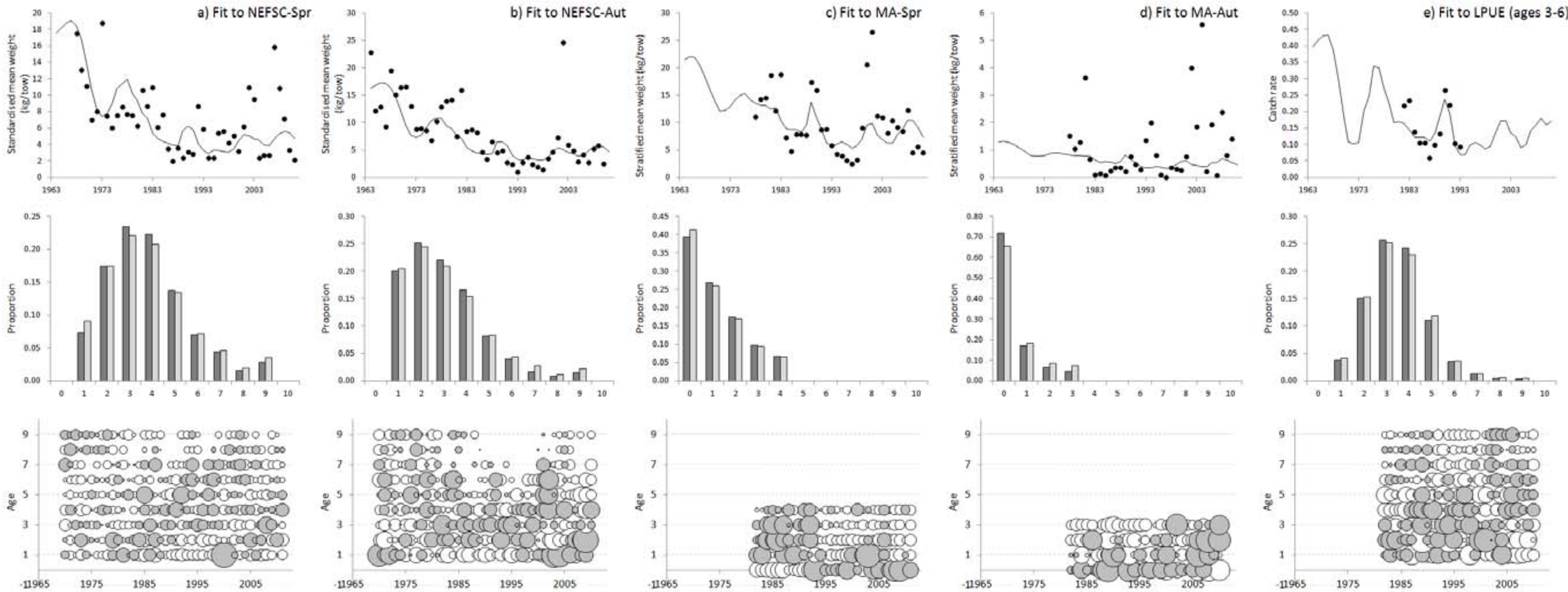

Fig. 2b: Fit to the abundance indices for **Base Case B (Beverton-Holt, *h*=0.98)** (top row) and to the survey and commercial catch-at-age data. The middle row plots compare the observed and predicted CAA as averaged over all years for which data are available, while the bottom row plots show the standardised residuals, with the size (area) of the bubbles being proportional to the magnitude of the corresponding standardised residuals. For positive residuals, the bubbles are grey, whereas for negative residuals, the bubbles are white.

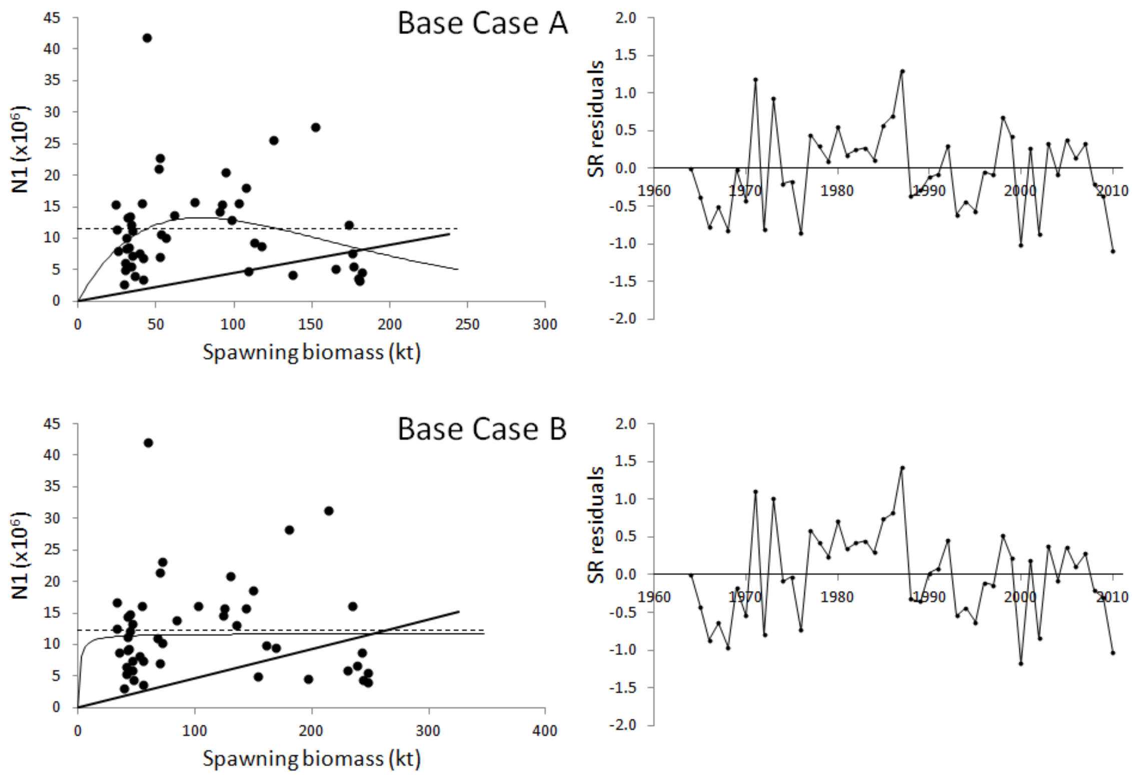

Fig. 3: The estimated stock-recruitment curves and time series of estimated recruitment residuals for the two Base Cases. The (arithmetic) average recruitment over the whole period is also shown (dashed line), as well as the replacement line.

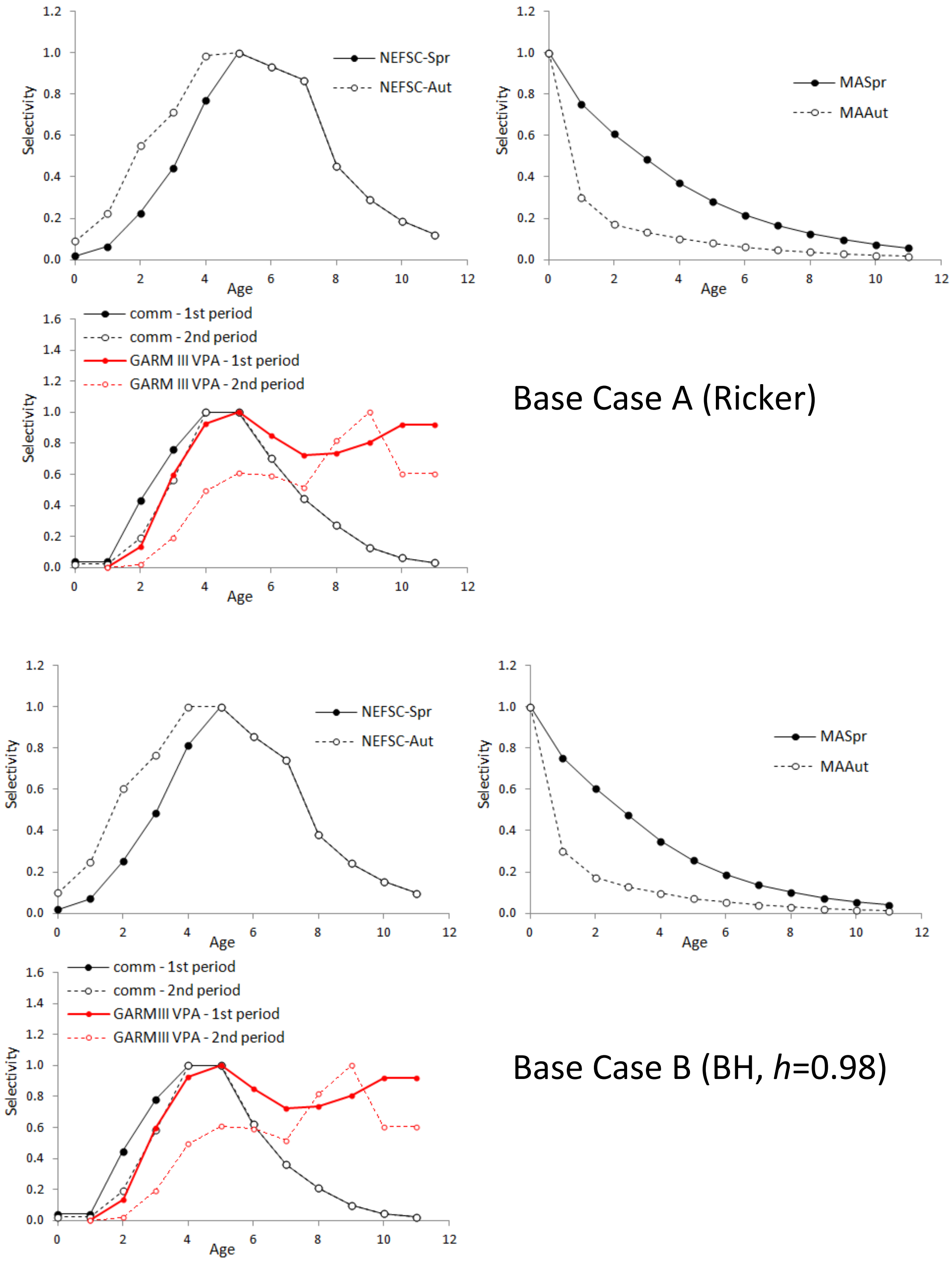

Fig. 4: Survey and commercial selectivities-at-age estimated for **Base Case A (Ricker)** and **Base Case B (Beverton-Holt, *h*=0.98)**.

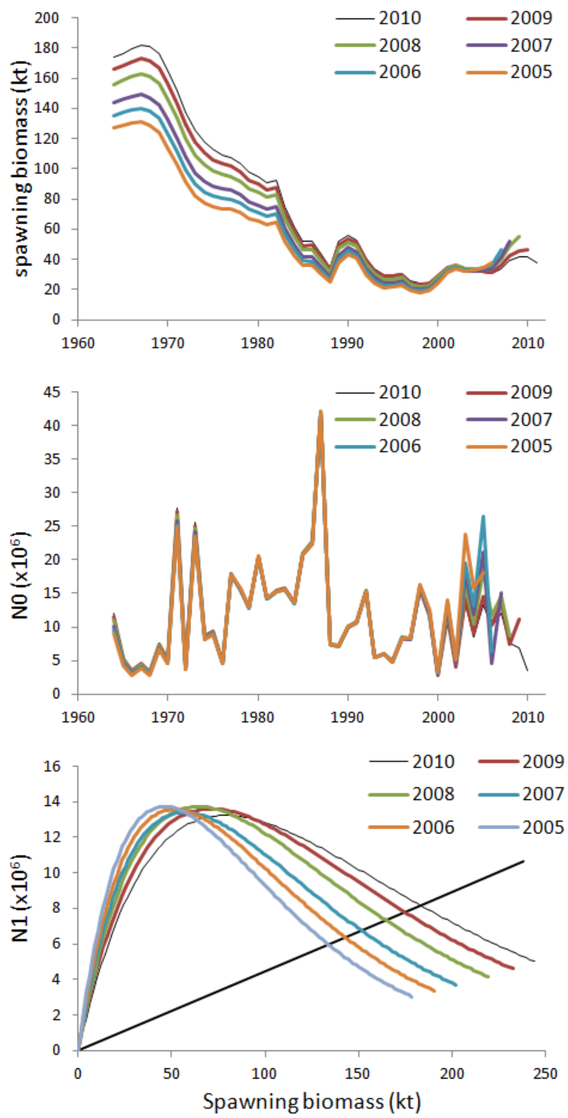

Fig. 5a: Retrospective analysis for **Base Case A (Ricker)** for spawning biomass, recruitment and stock recruitment curve.

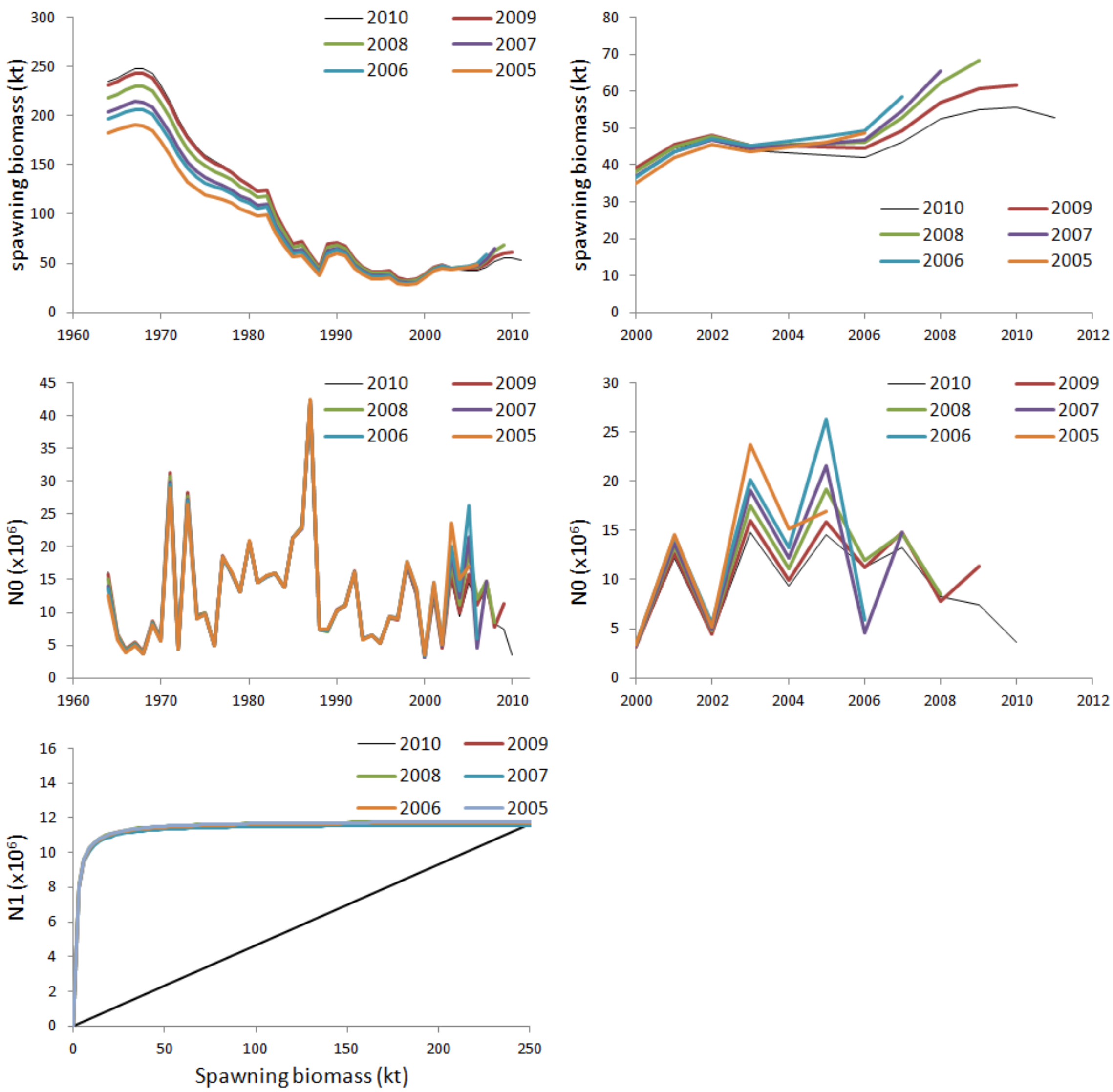

Fig. 5b: Retrospective analysis for **Base Case B (Beverton-Holt, *h*=0.98)** for spawning biomass, recruitment and stock recruitment curve.

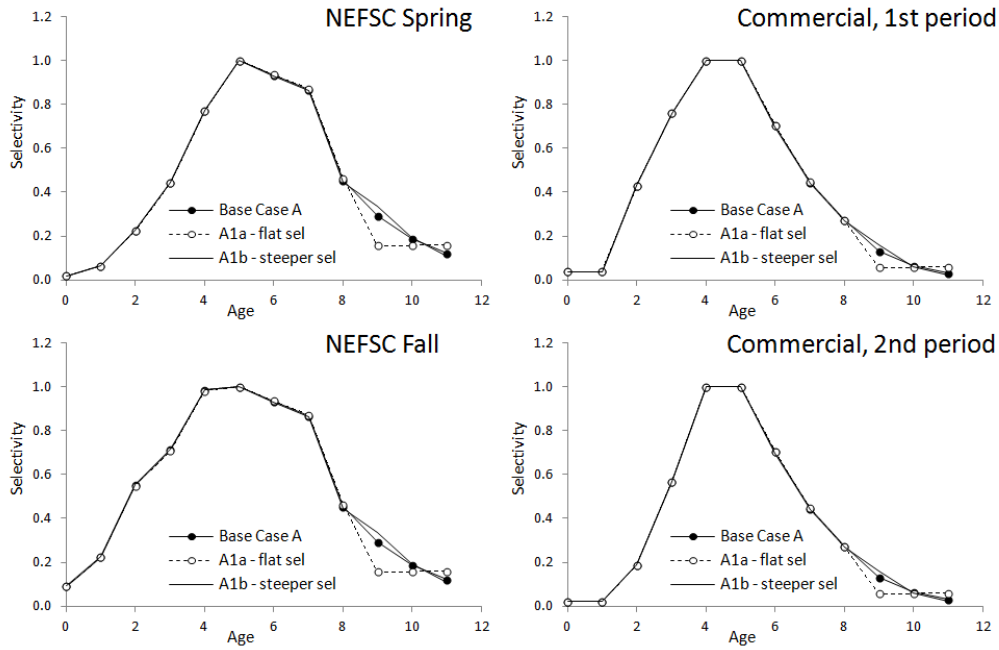

Fig. 6: NEFSC and commercial selectivities-at-age for Base Case A (Ricker), Case A1a (with flat selectivity for ages above 9) and Case A1b (with double the slope for ages 9 and above).

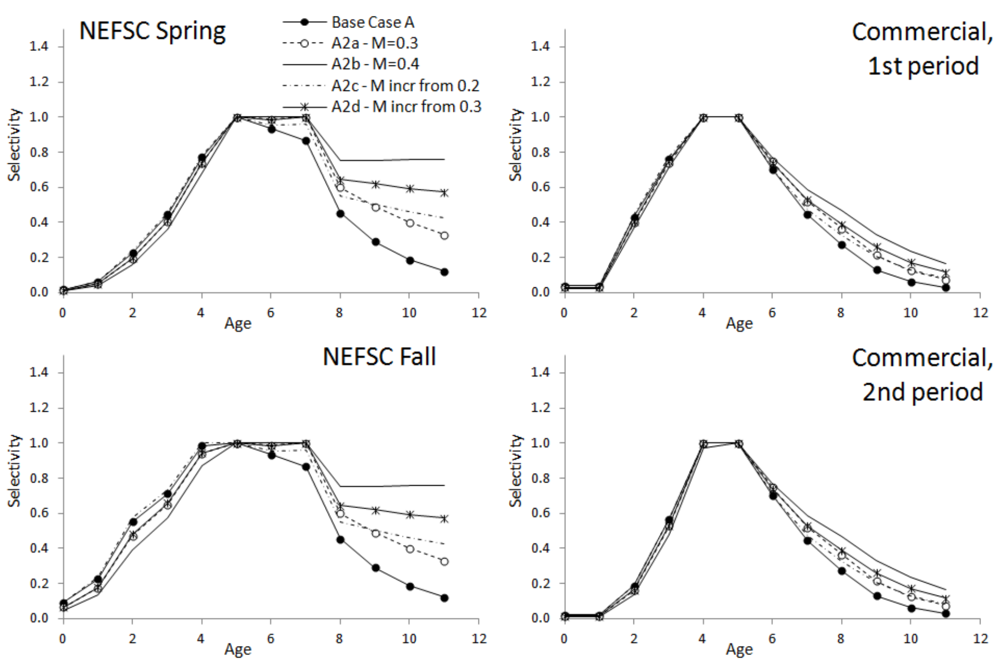

Fig. 7: NEFSC and commercial selectivities-at-age for Base Case A (Ricker), Case A2a (M=0.3), Case A2b (M=0.4), Case A2c (M increasing from 0.2) and Case A2d (M increasing from 0.3).

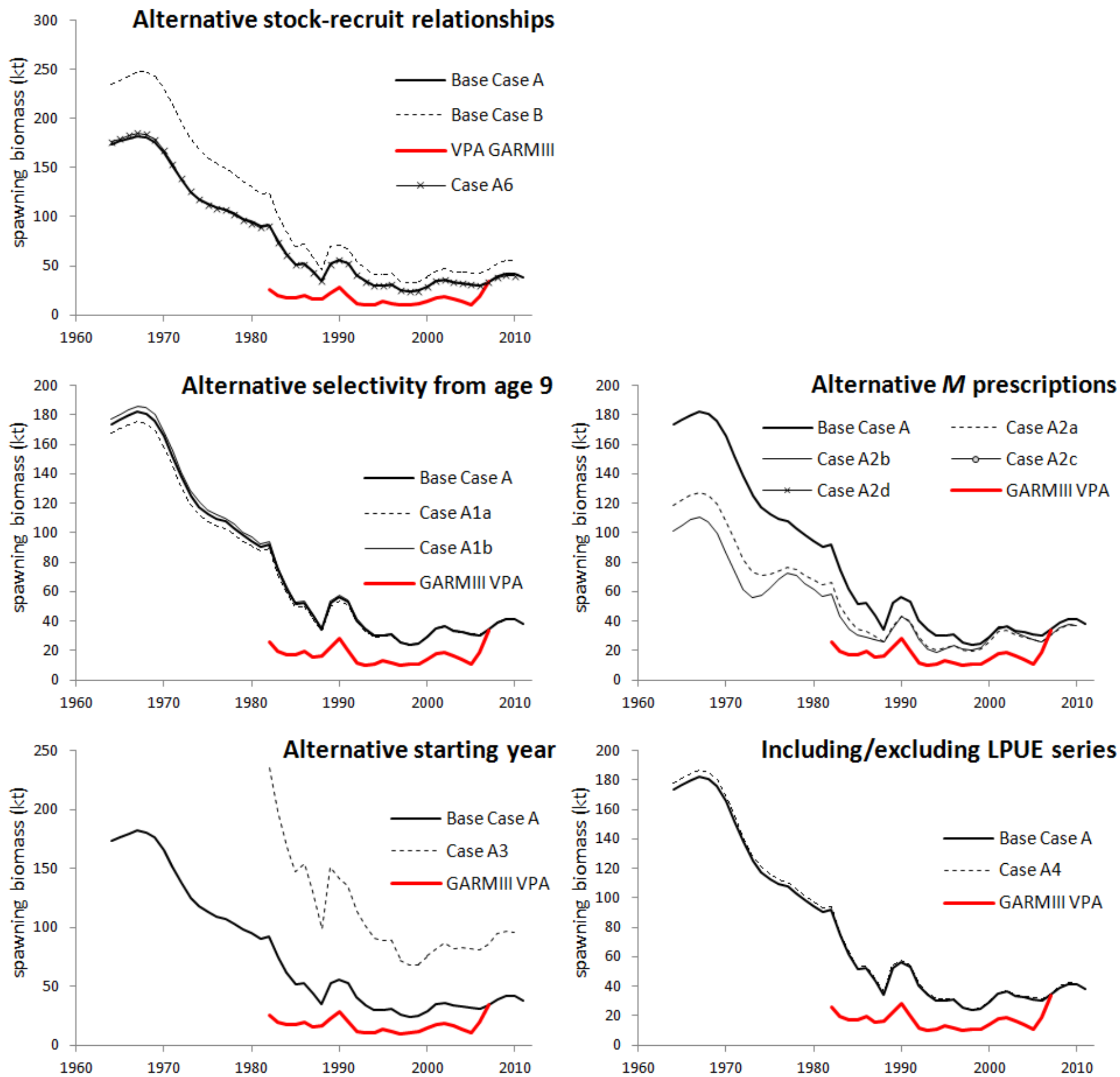

Fig. 8: Comparison spawning biomass trajectories for different variants of Base Case A (Ricker) (unless otherwise indicated).

APPENDIX A – Data

Table A1: Total catch (incl. USA, DWF and recreational landings, and discards) (thousand metric tons) of Atlantic cod from the Gulf of Maine (NAFO Division 5Y), 1964-2010 (Michael Palmer, pers. commn).

Year	Total catch	Year	Total catch	Year	Total catch
1964	3.242	1980	12.515	1996	8.250
1965	3.759	1981	17.882	1997	6.121
1966	4.225	1982	18.443	1998	4.968
1967	5.824	1983	17.494	1999	3.883
1968	6.137	1984	13.708	2000	6.961
1969	8.155	1985	15.807	2001	10.010
1970	7.961	1986	13.681	2002	8.366
1971	7.475	1987	13.772	2003	8.314
1972	6.927	1988	11.243	2004	7.072
1973	6.138	1989	14.623	2005	6.845
1974	7.550	1990	20.959	2006	4.996
1975	8.788	1991	22.273	2007	6.448
1976	9.894	1992	12.961	2008	8.818
1977	11.993	1993	10.993	2009	9.918
1978	11.890	1994	9.727	2010	11.392
1979	10.972	1995	8.190		

Table A2: Mean weight-at-age (kg) at the beginning of the year for the Gulf of Maine cod stock. Values derived from aggregated commercial landings and discard mean weight-at-age data (mid-year) using procedures described by Rivard (1980) (Michael Palmer, pers. commn). Pre-1982, the 1982-1991 average mean weight-at-age is assumed.

	1	2	3	4	5	6	7	8	9	10	11+
1982	0.241	0.595	1.159	2.100	4.659	7.594	9.326	8.986	13.095	15.545	18.490
1983	0.054	0.500	1.112	1.891	3.121	5.548	6.560	9.944	10.522	12.116	18.745
1984	0.075	0.373	1.016	2.018	2.957	4.589	7.096	7.842	11.798	12.803	16.269
1985	0.015	0.405	0.908	2.010	3.546	4.642	6.865	9.670	11.098	13.499	14.287
1986	0.104	0.320	1.076	1.912	3.679	5.551	6.960	9.364	12.122	13.021	18.289
1987	0.028	0.408	0.783	2.257	3.561	5.938	8.167	9.520	11.824	13.851	15.981
1988	0.022	0.297	0.982	1.715	3.978	4.893	6.714	10.150	10.779	15.542	11.779
1989	0.027	0.292	0.888	2.167	3.169	5.571	6.912	8.682	13.075	14.593	24.532
1990	0.095	0.436	0.936	1.736	3.628	5.749	8.008	10.375	13.700	16.599	22.637
1991	0.070	0.446	1.074	1.678	2.837	5.668	9.038	11.457	13.354	9.656	24.937
1992	0.029	0.473	1.202	2.009	2.550	4.604	8.821	10.500	12.827	17.092	23.502
1993	0.047	0.190	1.242	1.683	3.420	4.064	7.395	12.208	12.264	15.359	23.790
1994	0.039	0.239	0.979	2.216	2.539	5.257	6.562	11.144	11.349	17.850	22.643
1995	0.052	0.279	0.919	1.973	3.899	4.682	8.518	10.009	15.767	14.765	23.025
1996	0.063	0.357	1.412	1.743	2.925	6.170	8.930	12.844	14.577	19.623	22.643
1997	0.050	0.401	1.419	2.360	2.536	3.927	8.252	11.932	14.994	16.970	17.822
1998	0.059	0.259	1.405	2.199	3.397	3.518	5.691	10.439	14.582	15.340	17.822
1999	0.045	0.342	1.162	2.212	3.095	4.728	5.239	8.306	12.198	17.158	17.822
2000	0.120	0.459	1.042	2.208	3.385	4.731	5.483	7.876	10.994	13.346	19.237
2001	0.097	0.451	1.262	2.344	3.776	5.021	6.471	6.933	9.071	14.807	17.323
2002	0.088	0.454	1.025	2.175	3.135	5.219	6.464	7.839	10.049	9.717	19.237
2003	0.092	0.339	1.021	1.690	2.881	3.997	6.826	8.018	9.636	11.483	15.409
2004	0.067	0.362	0.954	2.077	2.550	4.083	5.812	8.636	10.767	12.291	15.875
2005	0.061	0.249	0.822	1.629	3.277	3.761	5.674	7.538	10.158	13.191	15.653
2006	0.089	0.301	0.812	1.892	2.447	4.054	4.881	6.757	8.837	11.576	16.669
2007	0.124	0.450	0.921	1.774	3.006	3.726	5.038	6.330	8.734	10.990	15.478
2008	0.086	0.419	1.104	1.862	2.854	3.564	5.120	6.893	8.376	12.526	16.157
2009	0.171	0.482	1.232	2.234	2.855	3.600	4.660	6.697	9.148	10.008	15.550
2010	0.105	0.587	1.140	2.231	3.067	3.536	4.786	7.207	8.754	10.633	14.792
2011	0.115	0.448	1.042	1.998	2.846	3.696	4.897	6.777	8.770	11.147	15.729

Table A3: Mean weight-at-age (kg) of landings for the Gulf of Maine cod stock (Michael Palmer, pers. commn). Pre-1982, the 1982-1991 average mean weight-at-age is assumed.

	0	1	2	3	4	5	6	7	8	9	10	11+
1982	0.013	0.347	0.813	1.480	2.560	5.084	7.058	9.630	9.725	12.596	19.184	18.490
1983	0.024	0.226	0.721	1.520	2.415	3.806	6.055	6.097	10.268	11.386	11.655	18.745
1984	0.001	0.236	0.617	1.434	2.678	3.621	5.533	8.315	10.087	13.557	14.397	16.269
1985	0.039	0.210	0.694	1.337	2.818	4.694	5.951	8.517	11.245	12.210	13.442	14.287
1986	0.005	0.278	0.488	1.668	2.736	4.804	6.565	8.139	10.296	13.067	13.886	18.289
1987	0.004	0.160	0.601	1.257	3.054	4.634	7.340	10.159	11.136	13.580	14.681	15.981
1988	0.003	0.124	0.550	1.606	2.339	5.182	5.166	6.142	10.141	10.434	17.787	11.779
1989	0.046	0.248	0.689	1.433	2.925	4.294	5.990	9.247	12.272	16.858	20.410	24.532
1990	0.021	0.195	0.766	1.271	2.105	4.500	7.697	10.705	11.641	15.294	16.344	22.637
1991	0.014	0.236	1.020	1.506	2.216	3.825	7.139	10.613	12.261	15.318	6.097	24.937
1992	0.023	0.058	0.949	1.416	2.679	2.935	5.541	10.900	10.389	13.418	19.072	23.502
1993	0.021	0.095	0.624	1.625	2.002	4.367	5.628	9.869	13.673	14.478	17.580	23.790
1994	0.022	0.074	0.601	1.536	3.023	3.221	6.328	7.650	12.583	9.420	22.008	22.643
1995	0.027	0.124	1.049	1.404	2.535	5.028	6.806	11.466	13.096	19.756	23.143	23.025
1996	0.033	0.146	1.038	1.902	2.164	3.374	7.572	11.717	14.388	16.225	19.490	22.643
1997	0.017	0.076	1.103	1.941	2.928	2.973	4.570	8.993	12.150	15.625	17.749	17.822
1998	0.008	0.203	0.881	1.790	2.492	3.941	4.163	7.086	12.118	17.500	15.060	17.822
1999	0.052	0.247	0.577	1.532	2.733	3.845	5.671	6.593	9.736	12.279	16.823	17.822
2000	0.030	0.278	0.853	1.882	3.181	4.192	5.821	5.302	9.409	12.416	14.506	19.237
2001	0.045	0.316	0.733	1.866	2.919	4.482	6.014	7.193	9.066	8.745	17.660	17.323
2002	0.032	0.171	0.652	1.433	2.535	3.366	6.078	6.949	8.542	11.138	10.797	19.237
2003	0.038	0.264	0.671	1.600	1.994	3.273	4.746	7.667	9.252	10.870	11.838	15.409
2004	0.025	0.117	0.498	1.357	2.696	3.262	5.095	7.118	9.729	12.530	13.897	15.875
2005	0.027	0.148	0.531	1.356	1.955	3.984	4.337	6.319	7.983	10.605	13.887	15.653
2006	0.073	0.295	0.611	1.243	2.639	3.062	4.125	5.493	7.226	9.783	12.635	16.669
2007	0.027	0.211	0.686	1.389	2.531	3.424	4.535	6.153	7.296	10.557	12.346	15.478
2008	0.090	0.272	0.833	1.779	2.497	3.219	3.710	5.780	7.723	9.616	14.863	16.157
2009	0.039	0.326	0.854	1.824	2.804	3.266	4.027	5.852	7.760	10.836	10.416	15.550
2010	0.022	0.281	1.057	1.521	2.730	3.354	3.828	5.687	8.876	9.875	10.434	14.792

Table A4: Total (commercial and recreational landings and discards) catches-at-age Gulf of Maine cod stock (Michael Palmer, pers. commn).

	0	1	2	3	4	5	6	7	8	9	10	11+
1982	1808	604384	3499150	2513857	1540671	794109	71019	102795	77180	48690	33459	10286
1983	18159	853211	3093927	3084251	1247253	730251	468178	52010	64209	28230	14612	15399
1984	24361	514713	2790043	1834222	1691050	451364	227720	108806	9628	17357	14699	22357
1985	89337	705350	2538198	2757311	1203812	780874	174584	119006	53895	6935	18043	11560
1986	23683	1032858	2345768	2941201	1053806	293218	217172	51299	41956	28308	9585	14847
1987	134239	411857	2927128	1937451	1734650	372528	98107	93300	17601	19821	17828	5834
1988	4593	570450	2076629	2350071	1243161	464054	70444	26850	28340	3257	2438	4184
1989	57	238764	1787389	2833027	1760371	544653	92775	74199	9857	5102	9000	6199
1990	0	90582	1076507	6483137	2910324	572113	201988	31252	40497	10223	16034	17714
1991	4335	169329	663323	1128243	6040013	1094511	154784	59882	25983	8495	4918	2571
1992	31737	504148	1081520	1038092	533497	2281418	231280	81134	6116	4488	1042	0
1993	35427	152090	1009076	2601432	1106412	107014	508532	42862	11264	0	0	0
1994	15645	178210	459803	1949771	1354650	275031	67118	75606	28923	6606	384	1008
1995	15429	116830	495214	1729698	1379445	228097	30388	6521	18250	580	2228	0
1996	29423	67809	195030	763475	2207626	426963	37114	4076	510	1842	0	0
1997	1963	100835	220657	624914	497422	927500	76134	5569	2336	393	636	0
1998	1167	18095	312450	606514	710776	158216	216525	29101	5277	1495	762	0
1999	84	143650	265110	517169	401575	213197	64150	71717	13933	1077	0	0
2000	0	75351	1033746	795592	949361	196861	91537	13595	11903	0	0	0
2001	0	779	946041	1778274	882305	457045	120312	63078	9108	11061	1006	0
2002	0	42222	95128	800985	1359521	440721	182745	74107	34482	9807	10260	4123
2003	30493	105267	330145	318550	1041079	946944	226101	83541	32383	17064	6231	6988
2004	249	250342	233562	1136733	347019	522604	290933	74266	35409	17128	8735	3329
2005	1980	41531	526947	335370	1568473	103318	278459	117667	30690	19037	7361	8096
2006	358	42356	134136	768475	364617	562395	35380	84394	42393	14054	8145	6369
2007	625	19362	262867	615205	1289449	161287	249098	8007	19331	10519	5163	6390
2008	1008	31317	358033	1027961	942773	937019	102439	117842	4421	9283	6155	2218
2009	183	28319	263922	1012781	1400125	581070	367916	22513	33904	1133	4437	5022
2010	265	28983	344671	1138753	1488899	1046776	249124	88188	14319	5765	891	4355

Table A5: Standardized stratified mean numbers per tow at age and standardized mean weight (kg) per tow of Atlantic cod in NEFSC offshore spring research vessel bottom trawl surveys in the Gulf of Maine, 1968-2011(Michael Palmer, pers. commn).

	0	1	2	3	4	5	6	7	8	9	10	11+	Stratified mean wt/tow	CV
1968													17.480	(0.153)
1969													13.100	(0.329)
1970	0.000	0.159	0.124	0.053	0.098	0.290	0.475	0.589	0.073	0.045	0.076	0.210	11.089	(0.237)
1971	0.000	0.069	0.109	0.099	0.280	0.086	0.096	0.280	0.207	0.142	0.050	0.013	7.004	(0.211)
1972	0.053	0.300	0.153	0.499	0.208	0.205	0.052	0.083	0.119	0.300	0.027	0.059	8.031	(0.233)
1973	0.000	0.053	4.273	0.917	0.614	0.384	0.144	0.106	0.186	0.276	0.186	0.386	18.807	(0.415)
1974	0.164	0.311	0.081	1.534	0.177	0.231	0.082	0.000	0.064	0.038	0.089	0.131	7.419	(0.199)
1975	0.012	0.094	0.707	0.095	1.139	0.246	0.073	0.000	0.006	0.025	0.028	0.088	6.039	(0.249)
1976	0.000	0.052	0.253	1.114	0.150	0.870	0.131	0.056	0.038	0.000	0.036	0.081	7.556	(0.166)
1977	0.000	0.068	0.264	0.460	2.015	0.139	0.775	0.000	0.114	0.000	0.000	0.038	8.541	(0.208)
1978	0.000	0.070	0.083	0.297	0.383	0.764	0.084	0.226	0.013	0.108	0.000	0.022	7.697	(0.207)
1979	0.044	0.426	1.407	0.186	0.470	0.301	0.549	0.094	0.104	0.013	0.031	0.020	7.555	(0.176)
1980	0.070	0.037	0.500	0.436	0.123	0.294	0.226	0.337	0.000	0.105	0.026	0.000	6.232	(0.182)
1981	0.000	1.091	0.619	0.850	1.335	0.318	0.304	0.080	0.144	0.091	0.000	0.000	10.650	(0.205)
1982	0.014	0.357	1.040	0.498	0.737	0.848	0.083	0.135	0.000	0.040	0.010	0.000	8.616	(0.223)
1983	0.013	0.610	0.968	1.042	0.453	0.336	0.250	0.060	0.000	0.071	0.033	0.077	10.962	(0.225)
1984	0.000	0.151	1.309	0.987	0.853	0.229	0.047	0.090	0.000	0.000	0.000	0.000	6.143	(0.324)
1985	0.000	0.029	0.238	0.676	0.612	0.707	0.094	0.109	0.026	0.026	0.000	0.000	7.645	(0.223)
1986	0.000	0.537	0.259	0.767	0.218	0.075	0.046	0.038	0.000	0.000	0.000	0.018	3.476	(0.197)
1987	0.000	0.030	0.471	0.191	0.222	0.075	0.000	0.068	0.011	0.000	0.000	0.015	1.976	(0.314)
1988	0.029	0.719	0.926	0.791	0.283	0.205	0.099	0.036	0.020	0.020	0.000	0.000	3.603	(0.281)
1989	0.000	0.025	0.609	0.712	0.630	0.069	0.068	0.000	0.000	0.000	0.000	0.000	2.424	(0.207)
1990	0.000	0.009	0.233	1.325	0.669	0.076	0.032	0.018	0.000	0.000	0.000	0.000	3.077	(0.280)
1991	0.000	0.028	0.077	0.233	1.750	0.247	0.041	0.018	0.000	0.000	0.000	0.000	2.891	(0.240)
1992	0.000	0.050	0.247	0.223	0.248	1.368	0.213	0.073	0.000	0.012	0.000	0.000	8.627	(0.374)
1993	0.000	0.201	0.507	0.804	0.364	0.084	0.446	0.055	0.023	0.000	0.023	0.000	5.875	(0.347)
1994	0.000	0.015	0.316	0.407	0.201	0.083	0.053	0.142	0.009	0.027	0.018	0.000	2.428	(0.216)
1995	0.000	0.037	0.187	1.165	0.321	0.147	0.034	0.000	0.011	0.000	0.028	0.000	2.432	(0.257)
1996	0.000	0.057	0.022	0.586	1.355	0.385	0.060	0.000	0.000	0.000	0.000	0.000	5.427	(0.275)
1997	0.000	0.159	0.139	0.390	0.271	0.874	0.244	0.115	0.000	0.000	0.000	0.000	5.616	(0.192)
1998	0.000	0.018	0.228	0.359	0.513	0.143	0.408	0.021	0.020	0.000	0.000	0.000	4.180	(0.324)
1999	0.000	0.166	0.342	0.726	0.351	0.305	0.134	0.266	0.000	0.000	0.000	0.011	5.090	(0.320)
2000	0.026	1.173	0.737	0.438	0.485	0.099	0.092	0.011	0.022	0.000	0.000	0.000	3.211	(0.155)
2001	0.000	0.029	0.355	0.683	0.510	0.342	0.065	0.097	0.055	0.000	0.011	0.000	6.215	(0.327)
2002	0.000	0.340	0.045	0.548	1.584	0.606	0.342	0.185	0.057	0.017	0.000	0.000	10.934	(0.215)
2003	0.000	0.075	0.825	0.059	0.718	1.072	0.387	0.340	0.081	0.082	0.030	0.011	9.495	(0.368)
2004	0.000	0.136	0.045	0.230	0.116	0.208	0.213	0.011	0.011	0.010	0.000	0.000	2.412	(0.293)
2005	0.000	0.029	0.739	0.081	0.623	0.011	0.138	0.128	0.015	0.000	0.000	0.000	2.701	(0.248)
2006	0.028	0.184	0.237	0.434	0.049	0.197	0.023	0.126	0.069	0.000	0.015	0.000	2.702	(0.249)
2007	0.000	0.100	3.422	3.077	4.446	0.437	0.796	0.075	0.041	0.000	0.000	0.000	15.811	(0.540)
2008	0.000	0.079	1.165	3.930	1.582	1.099	0.053	0.082	0.000	0.000	0.000	0.000	10.823	(0.609)
2009	0.000	0.063	0.279	1.050	1.135	0.600	0.438	0.008	0.022	0.000	0.004	0.000	7.161	(0.491)
2010	0.000	0.059	0.279	0.335	0.197	0.229	0.113	0.043	0.016	0.010	0.005	0.010	3.336	(0.264)
2011	0.000	0.005	0.024	0.140	0.383	0.189	0.086	0.033	0.035	0.000	0.000	0.000	2.133	(0.201)

Table A6: Standardized stratified mean numbers per tow at age and standardized mean weight (kg) per tow of Atlantic cod in NEFSC offshore autumn research vessel bottom trawl surveys in the Gulf of Maine, 1964-2010 (Michael Palmer, pers. commn).

	0	1	2	3	4	5	6	7	8	9	10	11+	Stratified mean wt/tow	CV
1964	-	-	-	-	-	-	-	-	-	-	-	-	22.799	(0.496)
1965	-	-	-	-	-	-	-	-	-	-	-	-	12.089	(0.273)
1966	-	-	-	-	-	-	-	-	-	-	-	-	12.838	(0.227)
1967	-	-	-	-	-	-	-	-	-	-	-	-	9.313	(0.219)
1968	-	-	-	-	-	-	-	-	-	-	-	-	19.437	(0.198)
1969	-	-	-	-	-	-	-	-	-	-	-	-	15.154	(0.217)
1970	0.743	0.938	0.254	0.520	0.336	0.487	0.424	0.836	0.130	0.090	0.037	0.110	16.442	(0.248)
1971	1.334	0.207	0.224	0.190	0.607	0.444	0.509	0.222	0.280	0.193	0.031	0.121	16.529	(0.307)
1972	0.031	5.663	1.118	1.595	0.181	0.072	0.122	0.031	0.121	0.351	0.000	0.016	12.988	(0.199)
1973	0.638	0.327	2.146	0.179	0.540	0.191	0.055	0.018	0.039	0.182	0.122	0.016	8.764	(0.267)
1974	0.265	1.131	0.267	1.922	0.125	0.276	0.000	0.052	0.036	0.066	0.000	0.189	8.959	(0.201)
1975	0.006	0.223	3.028	0.139	2.354	0.250	0.105	0.020	0.000	0.000	0.000	0.018	8.619	(0.153)
1976	0.000	0.209	0.216	0.578	0.104	0.835	0.044	0.099	0.000	0.000	0.063	0.000	6.740	(0.214)
1977	0.000	0.046	0.446	0.456	1.151	0.133	0.604	0.024	0.083	0.021	0.061	0.048	10.199	(0.126)
1978	0.241	1.411	0.359	1.141	0.661	1.450	0.101	0.269	0.012	0.082	0.000	0.047	12.899	(0.151)
1979	0.000	0.364	0.617	0.131	0.696	0.319	0.754	0.056	0.135	0.000	0.053	0.018	13.927	(0.128)
1980	0.027	1.319	2.558	1.664	0.518	0.236	0.402	0.192	0.022	0.012	0.000	0.085	14.202	(0.153)
1981	0.010	0.581	0.399	0.469	0.509	0.092	0.081	0.081	0.099	0.000	0.028	0.000	7.533	(0.233)
1982	0.000	0.835	3.264	2.476	0.971	0.222	0.000	0.000	0.000	0.000	0.000	0.000	15.919	(0.670)
1983	0.000	0.305	0.905	0.757	0.267	0.250	0.219	0.000	0.000	0.000	0.018	0.065	8.416	(0.188)
1984	0.000	0.513	0.418	0.586	0.384	0.196	0.194	0.062	0.000	0.016	0.000	0.080	8.735	(0.334)
1985	0.218	0.445	0.917	0.627	0.201	0.246	0.064	0.000	0.034	0.070	0.000	0.000	8.264	(0.354)
1986	0.000	0.394	0.404	0.626	0.368	0.073	0.041	0.000	0.000	0.045	0.000	0.000	4.715	(0.228)
1987	0.128	0.570	1.388	0.586	0.198	0.125	0.000	0.000	0.000	0.000	0.000	0.000	3.394	(0.234)
1988	0.000	1.889	2.366	1.069	0.367	0.146	0.000	0.044	0.000	0.011	0.011	0.000	6.616	(0.232)
1989	0.000	0.145	2.468	1.458	0.283	0.138	0.053	0.000	0.009	0.000	0.000	0.000	4.535	(0.181)
1990	0.000	0.057	0.218	1.788	0.611	0.255	0.048	0.010	0.000	0.000	0.000	0.000	4.912	(0.204)
1991	0.009	0.144	0.151	0.230	0.621	0.075	0.000	0.023	0.000	0.000	0.000	0.000	2.782	(0.246)
1992	0.059	0.289	0.448	0.144	0.041	0.327	0.126	0.000	0.000	0.000	0.000	0.000	2.448	(0.243)
1993	0.031	0.210	0.575	0.361	0.017	0.000	0.038	0.000	0.000	0.000	0.000	0.000	1.003	(0.263)
1994	0.032	0.184	0.909	0.816	0.093	0.051	0.000	0.045	0.000	0.000	0.000	0.000	2.737	(0.292)
1995	0.008	0.068	0.308	1.226	0.304	0.082	0.011	0.000	0.000	0.000	0.000	0.000	3.665	(0.325)
1996	0.029	0.122	0.379	0.231	0.516	0.050	0.000	0.000	0.000	0.000	0.000	0.000	2.352	(0.249)
1997	0.000	0.297	0.091	0.165	0.168	0.151	0.000	0.000	0.000	0.000	0.000	0.000	1.872	(0.307)
1998	0.050	0.085	0.342	0.110	0.185	0.041	0.031	0.000	0.000	0.000	0.000	0.000	1.501	(0.287)
1999	0.025	0.432	0.375	0.590	0.244	0.122	0.019	0.000	0.000	0.000	0.000	0.000	3.505	(0.193)
2000	0.008	0.540	0.981	0.399	0.492	0.140	0.010	0.000	0.034	0.000	0.000	0.000	4.652	(0.332)
2001	0.018	0.000	0.171	0.720	0.478	0.356	0.124	0.092	0.000	0.023	0.000	0.000	7.324	(0.279)
2002	0.000	0.269	0.104	0.333	2.683	1.070	0.750	0.077	0.043	0.000	0.000	0.000	24.659	(0.686)
2003	0.542	0.461	0.186	0.216	0.518	0.451	0.071	0.062	0.000	0.011	0.000	0.011	5.988	(0.251)
2004	1.369	0.661	0.172	0.577	0.254	0.250	0.149	0.057	0.023	0.010	0.011	0.000	4.906	(0.214)
2005	0.034	0.153	0.378	0.078	0.456	0.023	0.090	0.082	0.023	0.021	0.000	0.000	2.897	(0.228)
2006	0.064	1.241	0.599	1.007	0.252	0.293	0.037	0.053	0.036	0.000	0.000	0.014	4.229	(0.188)
2007	0.011	0.136	0.863	0.395	0.496	0.023	0.067	0.000	0.000	0.000	0.000	0.000	2.714	(0.277)
2008	0.165	0.650	1.227	1.060	0.189	0.139	0.000	0.000	0.000	0.010	0.021	0.000	5.307	(0.285)
2009	0.020	0.660	2.096	0.314	0.277	0.045	0.035	0.000	0.000	0.000	0.000	0.000	5.845	(0.429)
2010	0.008	0.094	0.132	0.290	0.288	0.092	0.023	0.013	0.000	0.000	0.000	0.006	2.572	(0.304)

Table A7: Stratified mean catch per tow in numbers and weight (kg) of Atlantic cod in State of Massachusetts inshore spring bottom trawl surveys in territorial waters adjacent to the Gulf of Maine (Mass. Regions 4-5), 1978-2011 (Michael Palmer, pers. commn).

	0	1	2	3	4	5	6	7	8	9	10	11+	Stratified mean wt/tow	CV
1978													11.058	(0.138)
1979													14.276	(0.219)
1980													14.509	(0.128)
1981													18.689	(0.265)
1982	1.691	13.261	6.765	2.830	0.943	0.221	0.046	0.035	0.050	0.000	0.000	0.000	12.162	(0.178)
1983	0.718	34.471	14.940	2.775	1.641	0.151	0.081	0.073	0.000	0.000	0.000	0.000	18.746	(0.159)
1984	0.257	2.038	4.916	2.304	0.582	0.147	0.086	0.000	0.000	0.000	0.000	0.000	7.241	(0.250)
1985	1.319	1.517	2.828	2.205	0.449	0.038	0.000	0.100	0.000	0.000	0.000	0.000	4.765	(0.202)
1986	1.075	8.694	12.316	0.948	0.935	0.099	0.023	0.000	0.000	0.000	0.000	0.000	7.842	(0.369)
1987	0.725	8.325	4.795	2.903	0.182	0.154	0.053	0.000	0.000	0.070	0.000	0.000	7.866	(0.289)
1988	1.881	9.997	6.867	1.852	1.574	0.000	0.038	0.033	0.000	0.000	0.000	0.000	7.705	(0.231)
1989	0.265	21.496	22.947	6.879	0.497	0.113	0.048	0.000	0.000	0.000	0.000	0.000	17.346	(0.331)
1990	4.942	4.485	6.206	14.159	2.263	0.282	0.072	0.000	0.000	0.000	0.000	0.000	15.880	(0.342)
1991	0.355	5.208	2.778	1.717	3.323	0.307	0.012	0.000	0.000	0.000	0.000	0.000	8.730	(0.123)
1992	1.506	4.461	5.526	3.419	0.576	1.290	0.102	0.044	0.000	0.000	0.000	0.000	8.766	(0.321)
1993	80.115	2.739	6.197	2.248	1.171	0.101	0.087	0.000	0.000	0.000	0.000	0.000	5.866	(0.278)
1994	4.627	5.142	3.907	1.901	0.632	0.149	0.000	0.000	0.000	0.000	0.000	0.000	4.338	(0.250)
1995	11.998	5.890	2.153	2.689	0.583	0.050	0.000	0.000	0.000	0.000	0.000	0.000	3.994	(0.234)
1996	8.843	0.777	0.497	1.091	1.482	0.272	0.000	0.000	0.000	0.000	0.000	0.000	3.153	(0.309)
1997	12.445	2.917	0.967	0.948	0.200	0.380	0.030	0.000	0.000	0.000	0.000	0.000	2.505	(0.256)
1998	23.481	1.531	0.823	0.772	0.707	0.034	0.205	0.017	0.000	0.000	0.000	0.000	3.254	(0.475)
1999	143.000	11.967	2.248	2.279	0.706	0.645	0.075	0.126	0.013	0.000	0.000	0.000	8.998	(0.254)
2000	2.151	35.402	7.197	2.592	2.048	0.712	0.523	0.059	0.087	0.000	0.000	0.000	20.605	(0.447)
2001	25.987	0.084	4.560	4.812	3.375	2.145	0.516	0.258	0.106	0.000	0.000	0.000	26.446	(0.533)
2002	0.924	19.299	0.255	1.352	1.287	0.526	0.270	0.104	0.235	0.025	0.049	0.012	11.160	(0.404)
2003	0.000	15.767	6.834	0.444	1.968	0.909	0.185	0.068	0.014	0.025	0.000	0.014	10.986	(0.222)
2004	116.149	8.955	1.799	2.661	0.351	1.000	0.534	0.098	0.029	0.000	0.014	0.000	8.151	(0.258)
2005	179.479	5.274	4.243	0.864	1.963	0.302	0.706	0.252	0.094	0.085	0.000	0.000	10.402	(0.195)
2006	0.000	10.634	6.601	3.844	0.566	1.464	0.106	0.077	0.000	0.009	0.028	0.000	9.178	(0.180)
2007	49.323	4.211	2.907	2.220	1.980	0.344	0.527	0.033	0.031	0.000	0.000	0.000	8.432	(0.243)
2008	456.954	7.181	10.018	3.920	2.097	1.588	0.187	0.155	0.000	0.000	0.000	0.000	12.231	(0.220)
2009	466.098	8.588	2.610	1.558	1.056	0.409	0.168	0.000	0.028	0.000	0.000	0.000	4.490	(0.189)
2010	1.165	2.626	1.261	1.398	0.680	0.656	0.231	0.007	0.000	0.000	0.052	0.000	5.645	(0.471)
2011	55.378	0.347	0.895	0.604	1.114	0.436	0.212	0.077	0.000	0.000	0.000	0.000	4.519	(0.428)

Table A8: Stratified mean catch per tow in numbers and weight (kg) of Atlantic cod in State of Massachusetts inshore autumn bottom trawl surveys in territorial waters adjacent to the Gulf of Maine (Mass. Regions 4-5), 1978-2010 (Michael Palmer, pers. commn).

	0	1	2	3	4	5	6	7	8	9	10	11+	Stratified mean wt/tow	CV
1978													1.515	(0.555)
1979													1.052	(0.377)
1980													1.286	(0.345)
1981													3.638	(0.453)
1982	4.593	1.009	0.334	0.131	0.046	0.000	0.000	0.011	0.000	0.000	0.000	0.000	0.662	(0.700)
1983	1.317	0.300	0.043	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.094	(0.533)
1984	10.228	0.244	0.060	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.139	(0.416)
1985	2.479	0.337	0.042	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	(0.390)
1986	1.883	0.447	0.392	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.250	(0.803)
1987	312.050	1.072	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.353	(0.184)
1988	5.396	3.230	0.236	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.368	(0.429)
1989	3.877	0.099	0.138	0.008	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.222	(0.422)
1990	7.660	4.286	0.443	0.269	0.024	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.761	(0.440)
1991	5.019	1.916	0.462	0.013	0.060	0.013	0.000	0.000	0.000	0.000	0.000	0.000	0.485	(0.516)
1992	26.311	1.093	0.054	0.000	0.000	0.038	0.000	0.000	0.000	0.000	0.000	0.000	0.286	(0.314)
1993	49.322	1.618	0.387	0.148	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.358	(0.235)
1994	39.877	5.624	2.977	0.507	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.003	(0.783)
1995	2.809	1.203	0.350	0.288	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.810	(0.658)
1996	6.921	0.059	0.003	0.006	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.096	(0.375)
1997	1.429	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015	(0.404)
1998	3.248	0.644	0.332	0.071	0.039	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.363	(0.499)
1999	7.515	0.372	0.102	0.008	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.310	(0.454)
2000	0.046	0.383	0.198	0.036	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.272	(0.386)
2001	49.171	0.035	0.135	0.125	0.063	0.027	0.000	0.000	0.000	0.000	0.000	0.000	0.760	(0.552)
2002	0.913	1.126	0.046	0.326	0.269	0.335	0.166	0.086	0.034	0.000	0.000	0.000	3.996	(0.768)
2003	119.971	0.731	1.168	0.110	0.164	0.092	0.048	0.000	0.000	0.000	0.000	0.000	1.859	(0.446)
2004	40.322	14.121	0.650	1.428	0.248	0.624	0.211	0.016	0.000	0.000	0.000	0.000	5.582	(0.400)
2005	39.189	0.785	0.355	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.212	(0.389)
2006	1.609	3.947	1.217	0.514	0.074	0.101	0.043	0.000	0.000	0.000	0.000	0.000	1.940	(0.460)
2007	7.573	0.217	0.096	0.031	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.082	(0.613)
2008	0.899	3.300	2.382	0.645	0.151	0.172	0.000	0.000	0.000	0.000	0.000	0.000	2.380	(0.462)
2009	2.908	1.046	0.733	0.298	0.041	0.008	0.009	0.000	0.000	0.000	0.000	0.000	0.811	(0.416)
2010	0.209	0.446	0.639	0.486	0.171	0.034	0.037	0.000	0.000	0.000	0.000	0.000	1.400	(0.488)

Table A9: USA commercial LPUE index through 1993 for ages 2-6.

	000s/days fished
1982	0.218
1983	0.233
1984	0.139
1985	0.106
1986	0.106
1987	0.060
1988	0.099
1989	0.133
1990	0.266
1991	0.221
1992	0.103
1993	0.094

Table A10: Percentage of mature females for each age for the Gulf of Maine cod stock (Michael Palmer, pers. commn).

0	1	2	3	4	5	6	7	8	9	10	11+
0.026	0.094	0.287	0.610	0.859	0.959	0.989	0.997	0.999	1.000	1.000	1.000

Appendix B - The Statistical Catch-at-Age Model

The text following sets out the equations and other general specifications of the SCAA followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is then applied to minimize the total negative log-likelihood function to estimate parameter values (the package AD Model Builder™, Otter Research, Ltd is used for this purpose).

B.1. Population dynamics

B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,0} = R_{y+1} \quad \text{(B1)}$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M_a/2} - C_{y,a}\right) e^{-M_a/2} \qquad \text{for } 0 \le a \le m-2 \quad \text{(B2)}$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M_{m-1}/2} - C_{y,m-1}\right) e^{-M_{m-1}/2} + \left(N_{y,m}\, e^{-M_m/2} - C_{y,m}\right) e^{-M_m/2} \quad \text{(B3)}$$

where

$N_{y,a}$ is the number of fish of age *a* at the start of year *y* (which refers to a calendar year),

R_y is the recruitment (number of 0-year-old fish) at the start of year *y*,

M_a denotes the natural mortality rate for fish of age *a*,

$C_{y,a}$ is the predicted number of fish of age *a* caught in year *y*, and

m is the maximum age considered (taken to be a plus-group).

These equations reflect Pope's form of the catch equation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov form (Baranov, 1918) (for which catches are incorporated under the assumption of steady continuous fishing mortality). Pope's form has been used in order to simplify computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulations will be minimal.

B.1.2. Recruitment

The number of recruits (i.e. new 0-year old) at the start of year *y* is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by either a modified Ricker or a Beverton-Holt stock-recruitment relationship, allowing for annual fluctuation about the deterministic relationship:

for the modified Ricker:

$$R_y = \alpha B_y^{\mathrm{sp}} \exp\left[-\beta\left(B_y^{\mathrm{sp}}\right)^{\gamma}\right] e^{(\varsigma_y - (\sigma_R)^2/2)} \quad \text{(B4)}$$

where

and for Beverton-Holt:

$$R_y = \frac{\alpha B_y^{\text{sp}}}{\beta + B_y^{\text{sp}}} e^{(\varsigma_y - (\sigma_R)^2/2)} \qquad \text{(B5)}$$

where

α, β and γ are spawning biomass-recruitment relationship parameters,

ς_y reflects fluctuation about the expected recruitment for year *y*, which is assumed to be normally distributed with standard deviation σ_R (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

B_y^{sp} is the spawning biomass at the start of year *y*, computed as:

$$B_y^{\text{sp}} = \sum_{a=0}^{m} f_{y,a} w_{y,a}^{\text{strt}} \left[N_{y,a} e^{-M_a/12} - C_{y,a}/6 \right] e^{-M_a/12} \qquad \text{(B6)}$$

because spawning for the cod stocks under consideration is taken to occur two months after the start of the year and some mortality (natural and fishing) has therefore occurred,

where

$w_{y,a}^{\text{strt}}$ is the mass of fish of age *a* during spawning, and

$f_{y,a}$ is the proportion of fish of age *a* that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruitment relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass, K^{sp}, and the "steepness", *h*, of the stock-recruitment relationship, which is the proportion of the virgin recruitment that is realized at a spawning biomass level of 20% of the virgin spawning biomass. In the fitting procedure, both *h* and K^{sp} are estimated with γ being either fixed on input or estimated as well.

B.1.3. Total catch and catches-at-age

The total catch by mass in year *y* is given by:

$$C_y = \sum_{a=0}^{m} w_{y,a}^{\text{mid}} C_{y,a} = \sum_{a=0}^{m} w_{y,a}^{\text{mid}} N_{y,a} e^{-M_a/2} S_{y,a} F_y^* \qquad \text{(B7)}$$

where

$w_{y,a}^{\text{mid}}$ denotes the mass of fish of age *a* landed in year *y*,

$C_{y,a}$ is the catch-at-age, i.e. the number of fish of age *a*, caught in year *y*,

$S_{y,a}$ is the commercial selectivity (i.e. combination of availability and vulnerability to fishing gear) at age *a* for year *y*; when $S_{y,a} = 1$, the age-class *a* is said to be fully selected, and

F_y^* is the proportion of a fully selected age class that is fished.

The model estimate of the mid-year exploitable ("available") component of biomass is calculated by converting the numbers-at-age into mid-year mass-at-age (using the individual weights of the landed fish) and applying natural and fishing mortality for half the year:

$$B_y^{\text{ex}} = \sum_{a=0}^{m} w_{y,a}^{\text{mid}} S_{y,a} N_{y,a} e^{-M_a/2} (1 - S_{y,a} F_y^* / 2) \qquad \text{(B8)}$$

whereas for survey estimates of biomass in the beginning of the year (for simplicity spring and autumn surveys are treated as mid-year surveys):

$$B_y^{\text{surv}} = \sum_{a=0}^{m} w_{y,a}^{\text{strt}} S_a^{\text{surv}} N_{y,a} e^{-M_a/2} (1 - S_{y,a} F_y^* / 2) \qquad \text{(B9)}$$

where

S_a^{surv} is the survey selectivity for age *a*, which is taken to be year-independent.

B.1.4. Initial conditions

As the first year for which data (even annual catch data) are available for the cod stock considered clearly does not correspond to the first year of (appreciable) exploitation, one cannot necessarily make the conventional assumption in the application of ASPM's that this initial year reflects a population (and its age-structure) at pre-exploitation equilibrium. For the first year (y_0) considered in the model therefore, the stock is assumed to be at a fraction (θ) of its pre-exploitation biomass, i.e.:

$$B_{y_0}^{\text{sp}} = \theta \cdot K^{\text{sp}} \qquad \text{(B10)}$$

with the starting age structure:

$$N_{y_0,a} = R_{\text{start}} N_{\text{start},a} \qquad \text{for } 1 \le a \le m \qquad \text{(B11)}$$

where

$$N_{\text{start},1} = 1 \qquad \text{(B12)}$$

$$N_{\text{start},a} = N_{\text{start},a-1} e^{-M_{a-1}} (1 - \phi S_{a-1}) \qquad \text{for } 2 \le a \le m-1 \qquad \text{(B13)}$$

$$N_{\text{start},m} = N_{\text{start},m-1} e^{-M_{m-1}} (1 - \phi S_{m-1}) \Big/ (1 - e^{-M_m} (1 - \phi S_m)) \qquad \text{(B14)}$$

where ϕ characterises the average fishing proportion over the years immediately preceding y_0.

B.2. The (penalised) likelihood function

The model can be fit to (a subset of) CPUE and survey abundance indices, and commercial and survey catch-at-age data to estimate model parameters (which may include residuals about the stock-recruitment function, facilitated through the incorporation of a penalty function described below). Contributions by each of these to the negative of the (penalised) log-likelihood (- $\ell\text{n}L$) are as follows.

B.2.1 LPUE relative abundance data

The likelihood is calculated assuming that an observed CPUE abundance index for a particular fishing fleet is log-normally distributed about its expected value:

$$I_y = \hat{I}_y \exp(\varepsilon_y) \quad \text{or} \quad \varepsilon_y = \ell n(I_y) - \ell n(\hat{I}_y) \qquad \text{(B15)}$$

where

I_y is the LPUE abundance index for year *y* for ages 2 to 6,

$\hat{I}_y = \hat{q}\ \widehat{N}_y^{\mathrm{ex}}$ is the corresponding model estimate, where $\widehat{N}_y^{\mathrm{ex}}$ is the model estimate of exploitable resource numbers for ages 2 to 6, given by

$$N_y^{\mathrm{ex}} = \sum_{a=2}^{6} S_{y,a} N_{y,a} e^{-M_a/2}(1 - S_{y,a} F_y^* / 2) \qquad \text{(B16)}$$

$\hat{q}$ is the constant of proportionality (catchability) for the LPUE abundance series, and

ε_y from $N\left(0,(\sigma_y)^2\right)$.

The contribution of the LPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell n L^{\mathrm{LPUE}} = \sum_y \left\{ \ell n \left(\sqrt{(\sigma_y^2 + \sigma_{Add}^2)} \right) + (\varepsilon_y)^2 / \left[2(\sigma_y^2 + \sigma_{Add}^2) \right] \right\} \qquad \text{(B17)}$$

where

σ_y is the standard deviation of the residuals for the logarithm of index *i* in year *y* (which is input), and

σ_{Add} is the square root of the additional variance for the LPUE abundance series, which is estimated in the model fitting procedure, with an upper bound of 0.5.

The catchability coefficient q^i for CPUE abundance index *i* is estimated by its maximum likelihood value:

$$\ell n\, \hat{q}^i = 1/n_i \sum_y \left(\ln I_y^i - \ln \hat{B}_y^{\mathrm{ex}} \right) \qquad \text{(B18)}$$

B2.2. Survey abundance data

In general, data from the surveys are treated as relative abundance indices in exactly the same manner to the CPUE series above, with survey selectivity function S_a^{surv} replacing the commercial selectivity $S_{y,a}$. Account is also taken of the time of year when the survey is held. For these analyses, selectivities are estimated as detailed in section B.4.2 below.

B.2.3. Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an "adjusted" lognormal error distribution is given by:

$$-\ell n L^{\mathrm{CAA}} = \sum_y \sum_a \left[\ell n \left(\sigma_{\mathrm{com}} / \sqrt{p_{y,a}} \right) + p_{y,a} \left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a} \right)^2 / 2(\sigma_{\mathrm{com}})^2 \right] \qquad \text{(B19)}$$

where

$p_{y,a} = C_{y,a} / \sum_{a'} C_{y,a'}$ is the observed proportion of fish caught in year *y* that are of age *a*,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a'} \hat{C}_{y,a'}$ is the model-predicted proportion of fish caught in year *y* that are of age *a*,

where

$$\hat{C}_{y,a} = N_{y,a}\, e^{-M_a/2}\, S_{y,a}\, F_y \tag{B20}$$

and

$\sigma_{\rm com}$ is the standard deviation associated with the catch-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{\rm com} = \sqrt{\sum_y \sum_a p_{y,a} \left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a}\right)^2 / \sum_y \sum_a 1} \tag{B21}$$

The log-normal error distribution underlying equation (B19) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation (B19), for which the summation over age *a* is taken from age $a_{\rm minus}$ (considered as a minus group) to $a_{\rm plus}$ (a plus group).

B.2.4. Survey catches-at-age

The survey catches-at-age are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation (B19)) where:

$p_{y,a} = C^{\rm surv}_{y,a} / \sum_{a'} C^{\rm surv}_{y,a'}$ is the observed proportion of fish of age *a* in year *y*,

$\hat{p}_{y,a}$ is the expected proportion of fish of age *a* in year *y* in the survey, given by:

$$\hat{p}_{y,a} = S^{\rm surv}_a N_{y,a} \Big/ \sum_{a'=0}^{m} S^{\rm surv}_a N_{y,a} \quad \text{for begin-year surveys.} \tag{B22}$$

B.2.5. Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed and serially correlated. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell nL^{\mathrm{pen}} = \sum_{y=y_1+1}^{y_2} \left[\left(\frac{\lambda_y - \rho\lambda_{y-1}}{\sqrt{1-\rho^2}} \right)^2 \Big/ 2\sigma_{\mathrm{R}}^2 \right] \tag{B23}$$

where

$\lambda_y = \rho\lambda_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y$ is the recruitment residual for year *y*, which is estimated for year y_1 to y_2 (see equation (B4)),

ε_y from $N\left(0,(\sigma_R)^2\right)$,

σ_{R} is the standard deviation of the log-residuals, which is input, and

ρ is the serial correlation coefficient, which is input.

In the interest of simplicity, equation (B23) omits a term in λ_{y_1} for the sensitivity when serial correlation is assumed ($\rho \neq 0$), which is generally of little quantitative consequence to values estimated.

The analyses conducted in this paper have however all assumed $\rho = 0$.

B.3. Estimation of precision

Where quoted, 95% probability interval estimates are based on the Hessian.

B.4. Model parameters

B.4.1. Fishing selectivity-at-age:

The commercial fishing selectivity, S_a, as well as the fishing selectivities for the NEFSC offshore and Massachusetts inshore spring and autumn surveys, are estimated separately for ages a_{minus} to a_{plus}. The estimated decrease from ages a_{plus}-1 to a_{plus}. is assumed to continue exponentially to age 11+ if otherwise not specified (see Table below for a_{minus} to a_{plus}.).

The commercial selectivity is taken to differ over the 1893-1991 and 1992+ periods. The decrease from ages a_{plus}-1 to a_{plus}. however is taken to be the same throughout the period. The decision to incorporate a change after 1991 was made to remove non-random residual patterns in the fit to the commercial catch-at-age data if time-independence in selectivity was assumed.

Selectivity is taken to differ for the surveys, but the decrease from ages a_{plus}-1 to a_{plus}. is taken to be the same for both spring and autumn surveys.

B.4.2. Other parameters

Model plus group				
m	11			
Commercial CAA				
a_{minus}	1			
a_{plus}	9			
Survey CAA	NEFSC spr	NEFSC fall	MASS spr	MASS fall
a_{minus}	1	1	0	0
a_{plus}	9	9	4	3
Natural mortality:				
M	age independent or not, fixed			
Proportion mature-at-age:				
$f_{y,a}$	input, see Table A10			
Weight-at-age:				
w_y^{strt}	input, see Table A2			
w_y^{mid}	input, see Table A3			
Initial conditions (unless otherwise specified):				
θ	estimated (with upper bound of 0.95)			
ϕ	0.1			